*Electricity at high
pressures and frequencies*

Henry L. Transtrom
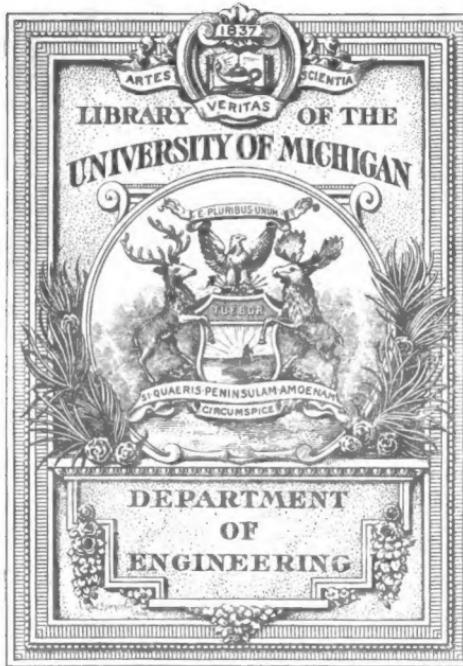
A PERSON CHARGED WITH HIGH POTENTIAL CURRENT.

ELECTRICITY AT HIGH PRESSURES AND FREQUENCIES

BY

HENRY L. TRANSTROM

PREFACE.

The trend of modern electrical work is in the direction of high potentials. It has been only a few years since a potential of 2,000 volts was considered unusually high, but at the present time we are using pressures as high as 110,000 volts, while that of 60,000 volts is not at all uncommon.

This is largely due to the locating of power plants at great distances from the district of distribution, requiring consequently high-potential transmission.

The author has also treated to a considerable extent of high-frequency currents, in order that the reader may not only have a clearer conception of its use in wireless telegraphy, telephony and electrotherapy, but also that he may more fully understand the peculiar surges produced on long transmission lines by lightning.

While the author does not believe that the use of high-frequency currents will supersede those of lower frequencies, yet such currents should be understood even in practical work.

With high-frequency currents transmission becomes difficult, owing to the enormous inductive reactance and capacity effects, which makes transmission over long distances impossible. Although currents of almost inappreciable quantity can be transmitted by means of electromagnetic waves, the current so transmitted is not sufficient for any practical use, notwithstanding popular opinion to the contrary.

This work is necessarily a brief treatise only of some of the principles and wonders of high-frequency currents at high and other potentials, owing to the limited space and the high technical knowledge required to completely fathom the problems of this as yet new field.

LIST OF ILLUSTRATIONS

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Electricity at High Pressures and Frequencies.

CHAPTER I.

Many electrical men who are engaged in practical work oftentimes become intensely interested in the wonders of currents of electricity at high pressures and frequencies, but find that the subject is often couched in such technical language that persons of only an ordinary education in the electrical science find that the subject is over their heads, or the other extreme may be that it is written in a more popular style, with little regard given to details, or the whys and wherefores, being, as it were, just skimmed over.

It is the object, therefore, of this book to bring to the mind of the practical electrician a lucid explanation of some of the apparently inexplicable effects of high pressure and frequency currents.

There are, no doubt, many of my readers who are quite familiar with the **elements** of electricity and magnetism as used in their respective branches of the profession; but, for the benefit of those who, although proficient in their field yet have not had the advantage of a thorough knowledge of the **principles** upon which the whole realm of the electrical science is founded, it would be well to go into a few of the fundamental principles.

For those who may think that much valuable time is lost in studying the simpler principles, it is sufficient for them to consider that it would be just as reasonable to expect a child to understand mathematics or grammar before it could read or write as to launch directly into the more complex actions of electricity at high pressures and frequencies.

Before going farther, it would be well to define what is meant by the expression electric " current." This can not be done absolutely because its true nature is unknown by man, but many of the **laws** which govern its manifestations are well known and perhaps best illustrated by analogy. The action of flowing water is in many ways similar to that of electricity, but it should not be followed too closely, for similarity does not mean identity. In the first place electricity does not flow, but its course of action is more **conveniently** described as though it did.

In electricity there is no transference of matter as there is in the analogy of the flow of water.

With this precaution in mind, assume that a length of pipe one inch in diameter and one hundred feet long is placed in a horizontal position, one end being connected with a large tank of water which has a pressure great enough to force the water at the rate of ten gallons a second.

By **doubling** the pressure twenty gallons will be forced out in the same length of time, the size and length of pipe remaining unchanged.

With the pressure left as it was originally but the **cross-section** of the pipe doubled, the amount of water would be doubled in the same length of time, or twenty gallons.

With the pressure and size of pipe the same as originally but the pipe only one-half as **long**, the water will flow twice as fast or twenty gallons in one second.

If the pressure, size and length are the same as

originally, but instead of a smooth pipe, one which is very **rough** inside is used or one which is filled with buckshot, the flowing of the water will be resisted, depending upon the obstruction or roughness, and the flow will be less than ten gallons per second.

If a place in the pipe is very weak, the pressure of the water will burst the pipe at that point. If the pressure is doubled and the pipe shortened to one-half its original length and the cross-section doubled and the pipe made with a smooth interior, the flow will be eight times as great as it was, or **eighty** gallons per second.

Therefore, in summing up the results obtained from our analogy, it is obvious that the flow is governed by the **pressure** of the water, the **length,** size and **quality** of resistance of the pipe.

The pressure in this illustration could be called the watermotive force, which has its counterpart in the electromotive force of the electric current. Electromotive force is called the **potential**; or, in popular language, the pressure.

The term potential was first used in the study of gravitation, but later Mr. George Green applied it to electromotive force, the symbol being E. M. F. or simply E. The flow of water compares with the flow of the electrical current, the symbol being C. or I.

The resistance of the pipe is similar to the electrical resistance of a wire or other conductor, and depends upon its length, its cross-section and the quality of its resistance, which resistance can be compared to the buckshot or roughness in the pipe, and also the **material** of which the conductor is composed.

When the electric current flows continuously in the same direction, it is said to be direct current and its symbol is D. C.

Allesandro Volta was the first man to invent an apparatus to produce a continuous flow of current.

He accomplished this result by means of his **voltaic pile,** which consisted of metallic plates stacked in a pile, adjacent plates being separated by acidulated silk cloth. In Fig. 1 the very light lines represent the silk cloth, the medium lines represent one kind of metal and the very dark lines the other kind. This was the only source of current known for some time, but it was not very efficient. Various inventors later produced what is called to-day a wet cell or battery.

Daniell (1790-1845) produced one which is called by his name. He was acquainted with Michael Faraday, and it is due to his association with Faraday that he was enabled to produce it.

In the use of the electric current, it is necessary to have **units** of measurement. The units may best be illustrated by another analogy which, although not entirely correct, serves the purpose quite well.

Assume an orifice one inch square in a tank filled with gas at one pound pressure. If it were possible for one cubic foot of gas at one pound pressure to pass out of the orifice at the speed of 1,728 inches per second, then the **volume** of the stream of gas at the orifice has its electrical counterpart in the practical unit called the **ampere.** As the speed of a gas varies with the pressure, it does not harmonize with the action of the electric current whose speed is practically **constant,** but we will assume, for the sake of clearness, that the speed of the gas escaping is 1,728 inches per second at any pressure, and that the speed of electricity is 186,000 miles per second.

If the pressure of the gas is **doubled,** it naturally forces twice as much gas per second through the same orifice as with one pound pressure, and with the **speed** the same, the density of the gas at the orifice would be doubled. That is exactly what happens to the electric current. When the pressure in volts is doubled, the

current's intensity is doubled in amperes if the resistance remains the same.

The quantity of electricity that can flow in one second, at an intensity of one ampere, is one **coulomb.** If the pressure of the current at this time is one

VOLTAIC PILE.

FIG. I.
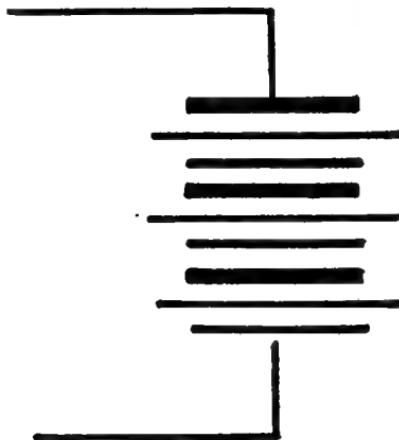
volt and the volume or intensity of the current flowing is one ampere, the resultant unit is one **Joule,** or one volt-coulomb, which is the quantity of work that one coulomb can do at a one-volt pressure.

The amount of work done per second, or rate of energy, is represented by the watt, which represents the activity of an electric current of one ampere at one volt pressure, which is the equivalent of 1-746 of a mechanical horse-power. One watt is equal to one joule per second. In the analogy the orifice one inch square is similar to the unit **ohm,** which is the unit of resistance. A law that was constructed by George

Simon Ohm (1789-1854) and therefore called **Ohm's law,** gives the relations of the pressure, current and resistance in a convenient form, as follows:

Current in amperes = Voltage ÷ Resistance I = $\dfrac{E}{R}$

This applies to direct currents; but with certain modifications can also apply to alternating currents.

ANALOGY OF THE FLOW OF GASES AND ELECTRIC CURRENT.

Element.	Speed per Second.	Quantity.	Intensity.	Pressure	Resistance.	Time.
Gas.	1,728 in.	1 cubic foot.	Stream of 1 sq. in. cross section.	1 pound.	Orifice 1 inch square.	1 sec.
Electric Current.	186,000 miles.	1 coulomb.	1 ampere.	1 volt.	1 ohm.	1 sec.

Lines of Force.

When a magnet is held near a piece of iron, it is attracted by the magnet, although the means used to do so are not visible.

To explain this invisible force which tends to pull the iron to the magnet, a theory has been constructed which assumes the existence of magnetic lines of force or more briefly " lines of force."

The paths or curves that these lines of force assume are plainly discerned by taking a glass plate or a paper, covering it with very fine iron filings sprinkled quite thinly and placing the glass directly on the magnet. The plate should be held perfectly level in order to obtain the best results.

In order to handle scientific subjects intelligently, a system of **units of measurement** has been constructed, in which system the unit of time is one **second,** the

unit of length one **centimeter,** the unit of weight is one gramme, etc.

One centimeter = 2-5 of an inch, approximately.

One gramme = 1-453.5 lb., or .00225 lb. approximately.

If one gramme is moved one centimeter, the work accomplished is called one **Erg,** which unit also represents the energy that had to be exerted to do the work. If one gramme is moved one centimeter in one second, the force required to do this is called one **Dyne.** The following illustrations give the actual sizes of these units.

One Gram.

METRIC UNITS.
FIG. 2.
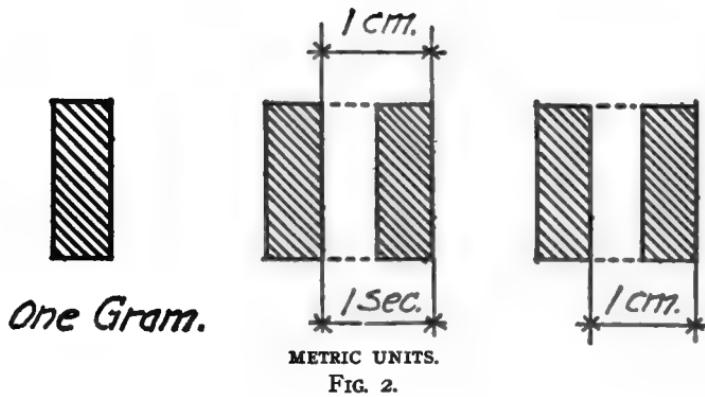
The force of gravity is 981.45 centimeters per second; therefore, the force required to **oppose** it in lifting one gram, one centimeter high, is 981.45 dynes. Thus the dyne can be expressed in weight, or 1-981.45 gramme.

One foot = 30.48 centimeters.

To lift one gramme one foot high requires 30.48 times 981.45 dynes of force, or 13,568,961 dynes. One horse-power is equal to 550 pounds lifted one foot high

per second, or 550 × 13,568,961 dynes or approximately 7,460,000,000 dynes of force.

The amount of work done is 7,460,000,000 ergs, which represents the **energy** expended.

The term " lines of force," as used in the illustration of the iron filings held in the vicinity of a magnet, is a term the great electrician, Michael Faraday (1791-1867) coined.   These lines of force decrease in number the greater the distance from the ends or poles of a magnet.

The two ends of a magnet are called its north and south poles respectively.   It is assumed that the lines of force leave the magnet at its north pole, marked N, and return at its south pole, marked S.

Poles of the same polarity repel **each other,** but unlike poles attract each other.

The magnetic field of a magnet is the space surrounding its poles.   If a magnet pole repels or attracts another magnet pole having the same number of lines of force issuing from it, with a force of one dyne when they are separated a distance of one centimeter, it is termed a **unit magnet pole.**   In other words the attrac-
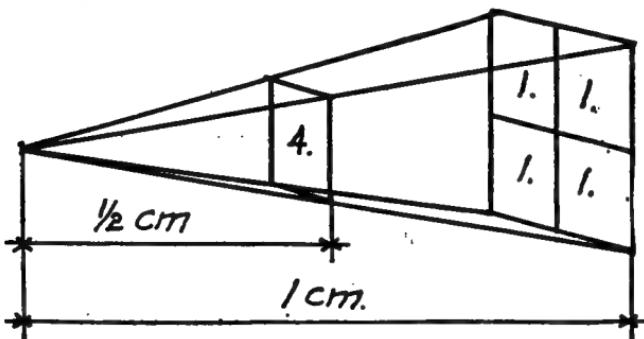
INVERSE SQUARES.
Fig. 3.

tion or pull is equal to a weight of 1·981 gramme when they are 1 c.m. apart.

The strength of a magnet decreases with the **square** of the distance; that is, in accordance with the law of inverse squares. In other words, a magnet is only one-fourth as strong one centimeter away from the magnet pole as it is one-half a centimeter away; and only one ninth as strong at one-third of a centimeter distance from the pole.

It is seen from illustration on page 8 that if one line of force fills each square one cm. away, that at one-half the distance all four lines are crowded into the same area formerly occupied by the one line; therefore at one-half the distance the strength is four times as great.

This rule is not quite true when the face of the pole is large compared with the distance.

The strength of the magnetic field of a unit magnet pole one centimeter away is one line of force per square centimeter, and is called a **unit magnetic field** (Fig. 4).

If we think of the lines of force of a unit magnet pole as issuing in all directions with an equal strength, then the total strength of such a pole is equal to 12.5664

UNIT MAGNETIC FIELD.
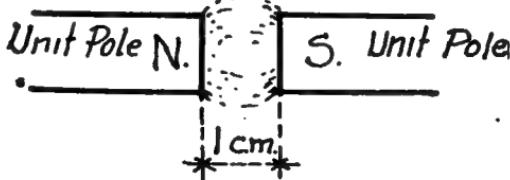
Fig. 4.

times the unit magnetic field, or one line of force = 12.5664 lines of force, which is called its **magnetic flux.**

The magnetic field, or flux, is expressed by the symbol Φ, which is a Greek letter pronounced phi.

The figures 3.1416 is the relation that the diameter has to the circumference of a circle, and is indicated by the Greek letter p, π pronounced pi.

12.5664 = 4 π or 4 $\times$ 3.1416, which is an equation often used.

4 π $\times$ radius ^{2} = the surface of a sphere, as the distance of one cm. is the radius in the unit magnet pole the surface would contain 4 π $\times$ 1^2 or 12.5664 sq. cm., and as each of these squares contain one line of force, the sum of all squares would be 12.5664 lines of force.

If the strength of the magnet is **doubled** the density is doubled, and in this case it would be two lines per sq. cm. $\mathcal{H}$ is a symbol for the number of lines of force per sq. cm.

So 4 π rad ^{2} $\mathcal{H}$ = 12.5664 lines of force in a unit pole, as $\mathcal{H}$ = 1 line of force.

4 π rad ^{2} $\mathcal{H}$ = Φ also.

Now the force in dynes at one cm. distance is unity in unit magnet pole, but if the **density** of $\mathcal{H}$ is doubled the force in dynes is also doubled, for 4 π rad ^{2} $\times$ 2 lines = 25.1328 lines = Φ.

If the Φ is divided by 4 π rad ^{2} the result = $\mathcal{H}$, or as it may be written

$$\frac{\Phi}{4 \pi \operatorname{rad}^2} = \mathcal{H}$$

In a bar or horseshoe magnet the lines of force must pass from its north pole through the air to its south pole.

If a piece of iron is used to connect the two poles, the number of lines issuing will be enormously increased, which shows that the lines can pass through the iron easier than through the air. **Magnetism** is analogous to the electric current in its attempt to choose the path of **least** resistance. The magnetic conductivity of matter is called its **permeability**. This

name was suggested by the much lamented Lord Kelvin.

Permeability represents the ease with which matter can be magnetized or demagnetized, and its symbol is μ, the Greek letter m, pronounced mu.

The permeability of the best qualities of iron is as high as 3,000 times that of air, which is taken as unity.

Tables have been computed giving the values of permeability of different grades of iron and steel.

Some substances have the peculiar property of being **repelled** by magnetism, and these are said to be diamagnetic, as, for instance, india-rubber, sulphur, glass, asbestos, etc. Michael Faraday found that glass was diamagnetized by suspending it by a silk thread between the poles of a powerful magnet. The glass adjusted itself diametrically at right angles to the poles, therefore the name.

Magnetic substances arrange themselves **parallel** to the lines of force, hence are called **paramagnetics,** as nickel, steel, iron, cobalt and antimony. Although some substances resist being magnetized more than others, there is no energy lost by its resistance after it is once magnetized. **Soft** iron or steel receive their magnetism readily, and lose it equally as readily, while **hard** iron and steel retain a considerable portion of their magnetism after the magnetizing influence is taken away. The magnetism retained is said to be its **residual** magnetism. Horseshoe and bar magnets are examples of this, and are called **permanent** magnets.

It is thought that originally the mass of iron consists of magnetized molecules promiscuously placed, but when the iron enters a magnetic field the molecules arrange themselves parallel to the lines of force in the field, all poles of the same polarity being, as it were, whirled in the same direction.

In very soft iron, which magnetizes readily and also demagnetizes readily, the molecules seem to have great

freedom to whirl, for as soon as the field is withdrawn the north and south poles of the molecules immediately whirl to unlike pole of similar molecules, so the visible effect is nil. In steel used as permanent magnets, this action is prevented to some extent because of the crowded condition of the molecules; so the metal remains polarized.   This seems reasonable, for in rapidly changing the magnetism in iron it becomes hot from friction occasioned by each molecule turning and rubbing against its neighbor.

Dr. Gilbert first discovered that if a hot piece of iron was hammered in a position parallel to the terrestrial or earth's magnetism that it acquired the property of magnetism, because the earth's magnetism was tending to turn the molecules parallel with its lines of force but could not do so until the hammering released them. It has also been noticed that a bar magnet loses part of its magnetism when held at right angles to the earth's magnetism and tapped with a hammer.

Page asserted that he heard a faint click of the molecules as he was magnetizing some iron.

It was once thought that electricity and magnetism were not related in any way; that it was merely a coincidence that usually both manifestations were present at the same time. Benjamin Franklin also entertained this opinion. But he was wrong, for magnetism can produce electricity, and conversely electricity produces magnetism.

This very important principle was discovered accidentally by a Danish professor, Hans Oerstad (1777-1851) of Copenhagen. At the conclusion of one of his lectures, Professor Oerstad placed a wire parallel and close over the needle of a compass, not knowing what to expect when the circuit was completed. The needle promptly assumed a position at almost right angles to the wire when the current flowed. He then reversed the current to see what would happen. The needle

turned half-way around, but still at right angles to the wire.

The secret was out, for it established the relation between magnetism and electricity. The reason for
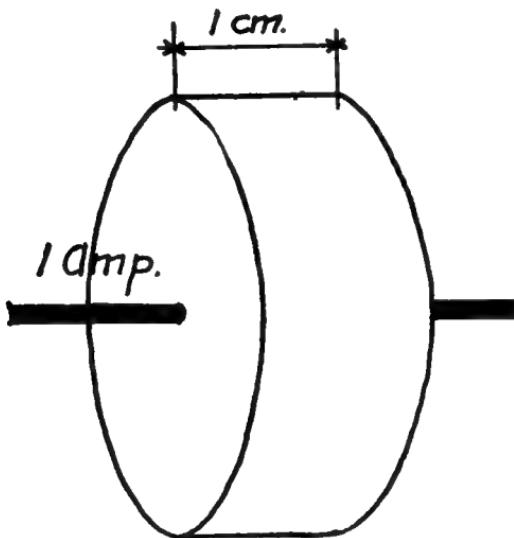
FIG. 5.

the peculiar action of the compass is that the needle adjusted itself parallel to the lines of force around the wire, for these are identical to those issuing from a permanent magnet.

If a current of electricity is passed through a conductor these lines of force are formed in concentric circles around it, which fact can be demonstrated by its effect on iron filings loosely scattered over a flat piece of paper held horizontally with a vertical wire passing through the center of the paper. The current should be continuous, that is, a direct current, D. C.

If a compass is held close to this conductor, the

north-seeking pole points in the direction of the rotation of the lines of force.

In the theory of electricity a substance named **ether** is assumed to exist in order to explain some electrical and magnetic phenomena. The ether mentioned here has no relation whatever to the drug of the same name. Ether is assumed to exist in the spaces between the atoms and molecules of all matter. All space is said to be filled with it. It is analogous to fine shot filling the spaces between apples in a barrel. Therefore, the ether would be almost entirely **unaffected** by an object in motion as the ether would pass through it as through a sieve.

But it is affected by electricity and magnetism.

Ether is considered as almost infinitely **elastic,** so lines of force are considered as a strain of the ether, and electricity a strain at right angles to that of the magnetism. The magnetic lines of force can be represented as an infinite number of rubber bands which are forced out in all directions **perpendicular** to the wire.

The absolute unit of current is 10 amperes, since it will produce a unit magnetic field of $\Phi = 12.5664$ lines around a conductor for every centimeter of its length, provided it is straight and the return conductor is very distant with **air** as the medium surrounding it (Fig. 5). Therefore, the practical unit of current is .1 of the absolute unit, which unit was found to be too large for convenience.

The lines of force issuing from a unit magnet pole are considered as moving in straight lines from a point in all directions, and the strength and density varied **inversely** as the square of the distance; but with the magnetic field the lines are in a different form. Surrounding a straight conductor the lines are in the form of **concentric circles,** whose ✕ varies inversely as the distance from the conductor. This can be clearly seen from the illustration on the opposite page (Fig. 6).
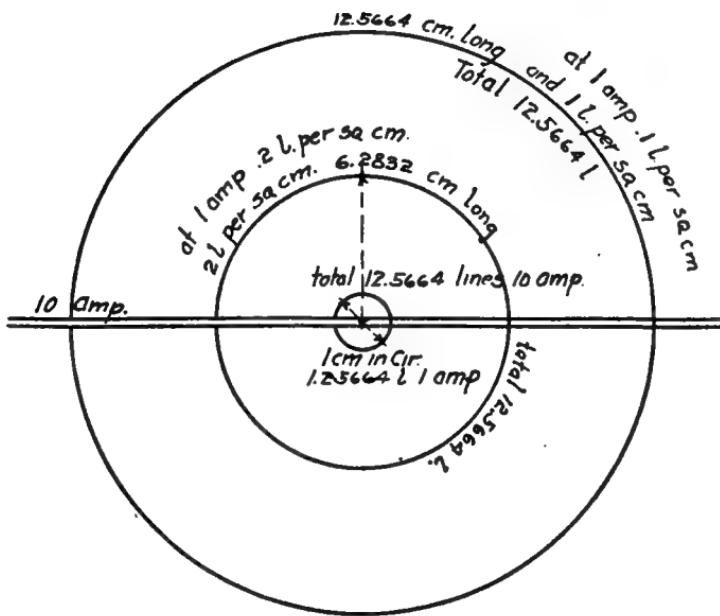
LINES OF FORCE SURROUNDING STRAIGHT
CONDUCTOR.

Fig. 6.

CHAPTER II.

As shown in the diagram of the lines of force surrounding a conductor (Fig. 6, previous page), it is seen that the flux Φ of a unit field around a conductor carrying unit current is distributed uniformly in concentric circles; therefore, the density $\mathcal{H}$ per square centimeter would be greater the less the circumference of the circles, and as the circumference equals $2\pi \times$ rad, it is evident that the radius must be two centimeters to form a circle of 12.5664 centimeters in circumference. From this it is seen that the unit field of flux Φ 12.5664 lines is distributed along 12.5664 centimeters, the density being, therefore, one line of force per square centimeter.

As the density decreases **directly** as the distance from the conductor, or **inversely** increases as the conductor is approached, it is obvious that at a distance of one centimeter from the conductor the density would be two lines per square centimeter.

The lines of force around a conductor can be considered as a closed **magnetic circuit,** the length of the circuit being the circumference of any circular line of force, which is 2π multiplied by the distance from the conductor, which distance is considered as the radius of the magnetic circuit.

As the density depends upon the length of the circuit, $\mathcal{H} = \dfrac{\Phi}{2\pi \text{ rad}}$; or, in the case of a unit field at unit distance, $\mathcal{H} = \dfrac{4\pi}{2\pi} = 2$ lines per square centimeter.
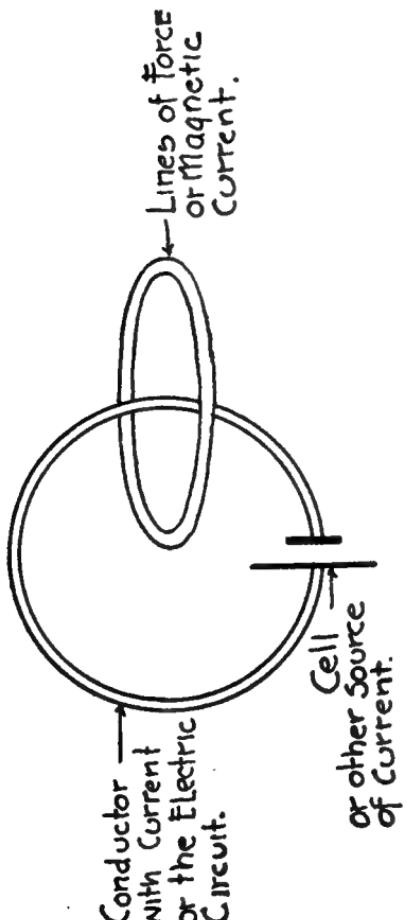
FIG. 7.

The current flowing through the conductor in this case is the absolute unit or 10 amperes.

Therefore, the flux around a conductor would only be .4 π or 1.25664 lines of force when one ampere is flowing, and the density $\mathcal{H}$ would only be .2 line at one centimeter and .1 line at 2 centimeters radius.

These results are only true when the return conductor is at a great distance. As the magnetic circuit completely surrounds the **electrical circuit** and inversely, as shown in Fig. 7, they are said to be interlinked.

When the current flowing in the electrical circuit is one ampere and that circuit, interlinking the magnetic circuit, consists of **one turn** of conductor, it is said to be one **ampere-turn.** See Fig. 8.

Thus, one ampere turn produces a density $\mathcal{H} =$ 1.25664 or .4 π lines of force, and $\Phi = A \times \mathcal{H}$ and is called one interlinkage with the magnetic field.

If the current is raised to 1,000 amperes through the same turn, the number of interlinkages would be 1,000 and also the magnetomotive force would be 1,000 ampere-turns, sometimes called Gilberts.

As the flux density would be increased by .4 π lines for every additional ampere-turn, the density $\mathcal{H} =$ .4 $\pi \times$ ampere-turns

If the electrical circuit is in the form of a helix consisting of a great number of turns, as shown in Fig. 9, and the volume of current small, the ampere-turns, and therefore the magnetic strength, would equal that produced by a heavy current flowing through an electrical circuit of few turns. For example, a coil having 1,000 turns of wire and having a current of one ampere flowing through it, would have precisely the same magnetic power as a coil of ten turns with a current of 100 amperes flowing through them.

It is plain, therefore, that ampere-turns are the

One turn
with one Ampere
One amp. Turn.
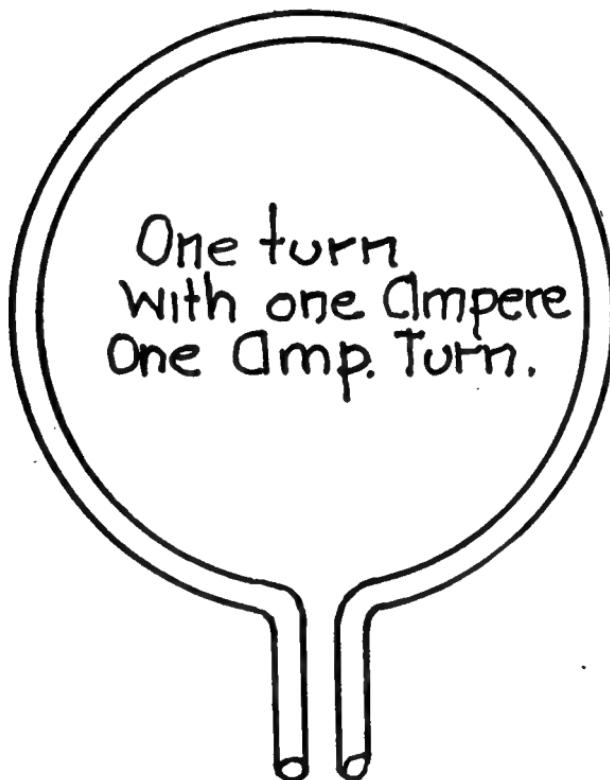
FIG. 8.

product of the number of turns by the current in amperes.

It was due to the genius of Andre Marie Ampere (1775-1836) that the conductor was formed into a helix to increase the strength of magnets.

In comparing the magnetic circuit with the electrical circuit, we find a great similarity.

The term **reluctance** is applied to the property of a magnetic circuit analogous to **resistance** in an electrical circuit, but is not identical because the resistance offered to the passage of current in an electrical conductor is the same whether the current is great or small, while the reluctance of a magnetic circuit is greater when the flux-density is great.

Flux in a magnetic circuit is a counterpart of **current** in an electric circuit.   Reluctance is the **converse**

4 Turns
with
$\frac{1}{4}$ Ampere
1 Ampere
Turn.
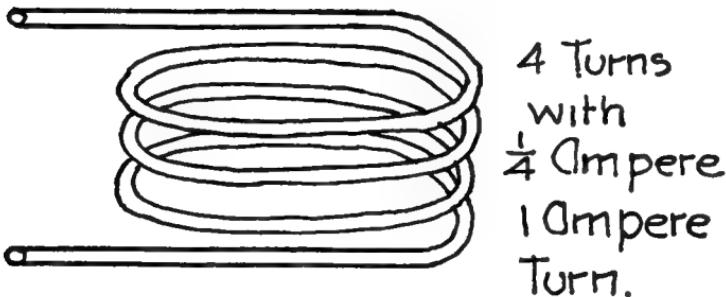
FIG. 9.

of permeability in a magnetic circuit, the same as resistance in an electrical circuit is the converse of conductance.

As we have here found that the resistance that a conductor presents to the passage of current is governed by its length, cross-section and material in its composition, so in like manner the reluctance in a magnetic circuit is governed by its length, cross-section and the magnetic properties or permeability of the material forming its path.

R is the **symbol** for reluctance.   In calculating the value of R when the magnetic circuit consists wholly

of iron, $R = \dfrac{\text{length in centimeters}}{\text{Cross-section} \times \text{permeability}}$, or substi-

tuting l for length and A for cross-section and μ for

permeability, we have $R = \dfrac{l}{A\,\mu}$

For example, we wish to find the **reluctance** of a magnetic circuit consisting of an iron core 50 c.m. long and 10 sq. c. m. in cross-section and with a per-

meability of 275, then $R = \dfrac{50}{10 \times 275}$ .01818, therefore

the reluctance is .01818.

If the circuit consists of several parts, as part of

iron and part of air, $R = \left\{ \dfrac{l_1}{A_1\,\mu_1} + \dfrac{l_2}{A_2\,\mu_2} + \text{etc.} \right\}$, that is,

it is equal to the **sum** of their individual reluctances.

The small index numbers 1, 2, etc., are the numbers of the parts of the circuit, showing what the length, cross-section and permeability are of those parts. As it is generally the ampere-turns that are desired, they may be found by multiplying the desired flux, as

$\Phi \times \dfrac{l}{A\,\mu} \div .4\,\pi =$ ampere-turns, or the magneto-

motive force; or, if the **core** consists of several parts,

$\Phi \times \left\{ \dfrac{l_1}{A_1\,\mu_1} + \dfrac{l_2}{A_2\,\mu_2} + \text{etc.} \right\} \div .4\,\pi =$ ampere-turns.

Magnetomotive force is analogous to electromotive force, and is indicated by ampere-turns, and its symbol is F.

As the permeability of iron and steel varies at different densities, charts have been prepared from observations, which show its value in different grades of iron and steel.

CURVES OF MAGNETIC INDUCTION
FOR WROUGHT IRON, CAST STEEL AND CAST IRON
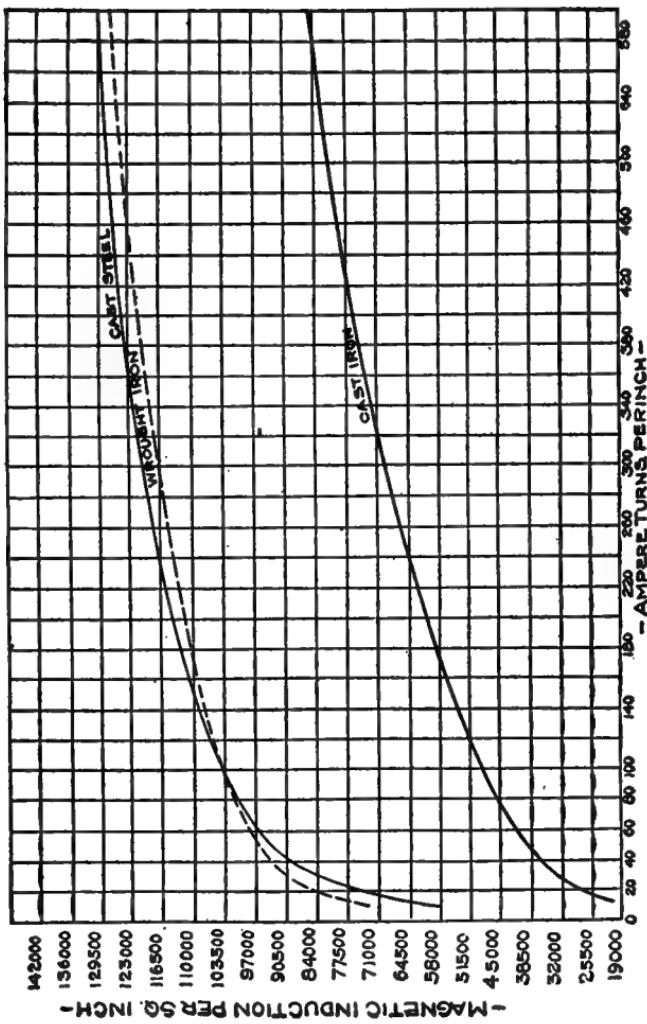
The foregoing is a chart showing the characteristics of various irons and steels.

The shape of the curve on the chart is called the **characteristic.**

The making of observations is called **plotting a curve.**

This method is also used to show the characteristics of currents, voltages, spark-lengths and many other uses, as shown below.

It is evident that when the curve reaches the point of **saturation,** the number of lines produced by one ampere-turn is very small, so that the average value of μ is much smaller than that at a low density.

The density of a field whose magnetic circuit is **air** is indicated by the symbol $\divideontimes$. If the density of a field whose path is through iron or steel or a substance with μ above 1, it is expressed by the symbol B, which indicates the number of lines per square centimeter.

As the permeability of **air** is about unity, and that

for **iron** much greater, $\mu = \dfrac{B}{\divideontimes}$

For example, if a magnetic field has a density of 40 lines per square centimeter in air, and the same number of ampere-turns produce 12,000 lines in iron,

$B = 12,000,\ \divideontimes = 40 \therefore \mu = \dfrac{12000}{40} = 300.$

If the value of the flux is desired, letting S denote the number of turns,

$$\Phi = \dfrac{4\,\pi\,I\,S}{10\,\dfrac{1}{A\,\pi}}$$

For example, a circular core has a cross-section of 10 square centimeters and a length of 80 centimeters, its permeability is 300 and is wound with 100 turns

with 6 amperes passing through it, then, by substitution,

$$\Phi = \frac{12.5664 \times 6 \times 100}{10 \times \dfrac{80}{10 \times 300}} = 28345 \text{ lines.}$$

The factor 10 in the equation is used to bring the current to **absolute** units.

If the magnetic circuit consists of several parts

$$\Phi = \frac{4\,\pi\,\text{I S}}{10 \left\{ \dfrac{l_1}{A_1\,\mu_1} + \dfrac{l_2}{A_2\,\mu_2} + \text{etc.} \right\}}$$

It is by means of coils of wire wound around **iron cores** that powerful magnets are produced.
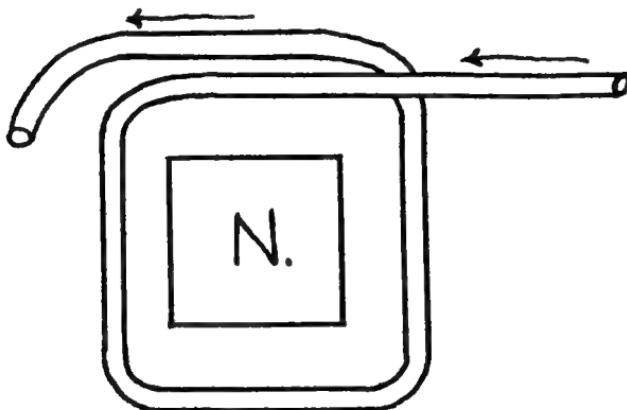
Fig. 10.

These magnets are called **electromagnets** to distinguish them from permanent magnets. They are many times more powerful than any permanent magnet can be made. They are generally made from **soft**

iron or steel, and used for motor field-magnets, electric-bells, magnetic-hoists, circuit-breakers, etc.

The **polarity** of an electromagnet is determined by **the direction** of the flow of current around it.

If you face one pole of a bar magnet and the current is rotating **counter clock-wise,** that pole toward you is the **north pole,** as shown in Fig. 10 and Fig. 11.
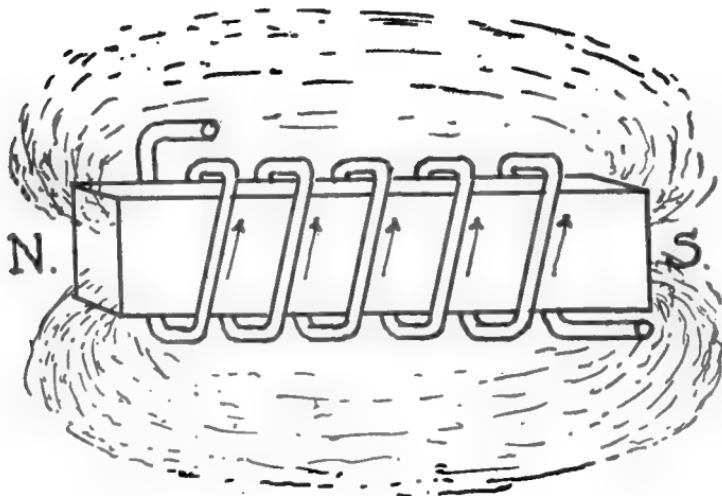
FIG. 11.

If two bar magnets lie parallel to one another with similar poles lying in the same direction, as shown in Fig. 12, the result would be **repulsion;** because, from along their whole lengths issue lines of force in the opposite direction, as shown in Fig. 12.

If the position of one of the pair is **reversed,** the lines would present a different appearance and would result in mutual attraction, therefore **dissimilar** poles attract, as shown in Fig. 13.
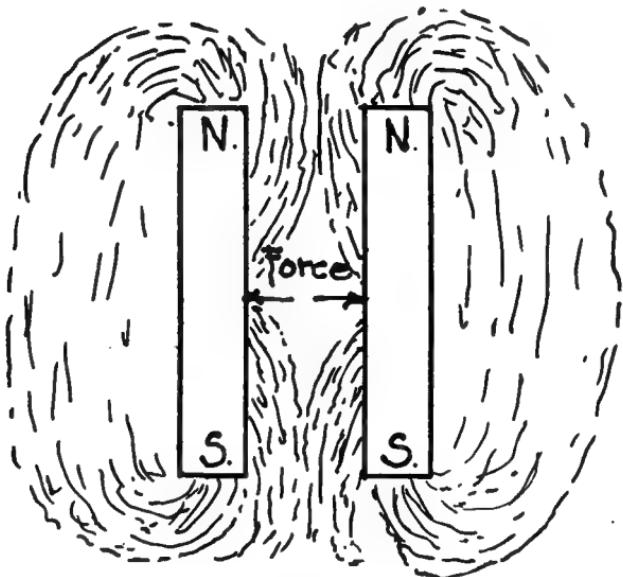
FIG. 12.

In observing the circles or chains of iron filings around a conductor, as shown in Fig. 14, passed through a glass plate or sheet of paper, it is noticed that the filings arrange themselves lengthwise and parallel to the lines of force, and that they are **oblong.** The reason for this is that **air** has a lower permeability than the iron, thus presenting a greater resistance to the lines.   In attempting to choose the path affording it the **least** resistance, the magnetic lines of force exert a force upon the particles to turn them and thereby **shorten** the air path of the flux.

A chain of iron filings lying within the path of the lines of force can then be considered as so many **bar magnets,** end to end, the north pole of one facing the

south pole of the next particle, and so on, until the north pole of the last particle faces the south pole of the first.

The action of **parallel wires** carrying current is shown in Figs. 15 and 16.
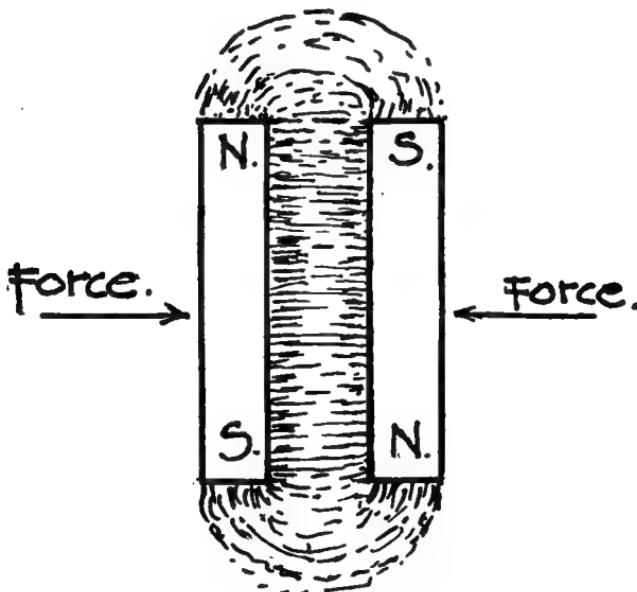
FIG. 13.

Parallel conductors carrying current in the **same** direction, Fig. 15, result in **attraction,** because their adjacent lines of force lie parallel.

If the currents in two parallel conductors flow in **opposite** directions, Fig. 16, **repulsion** occurs, since the fields in this case would be the reverse of the first; that is, **like** poles would be adjacent, causing repulsion as when the bar magnets faced in the same direction.
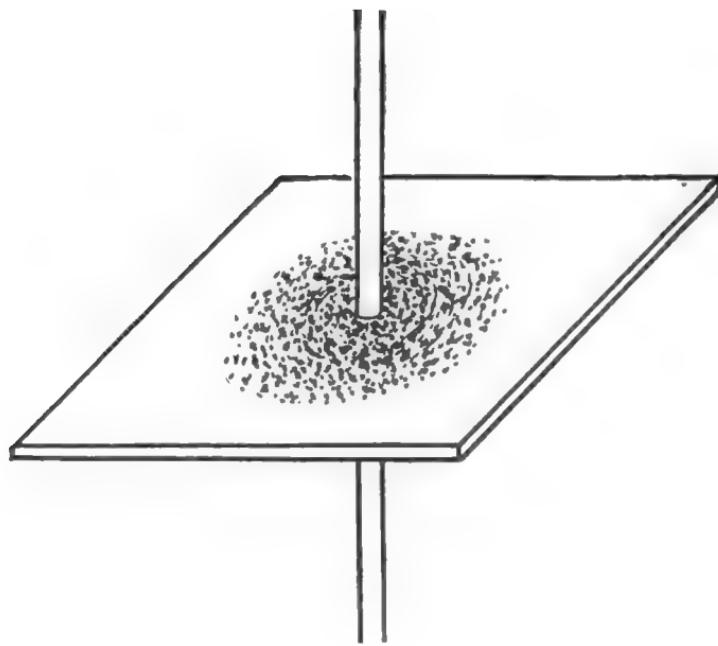
FIG. 14.
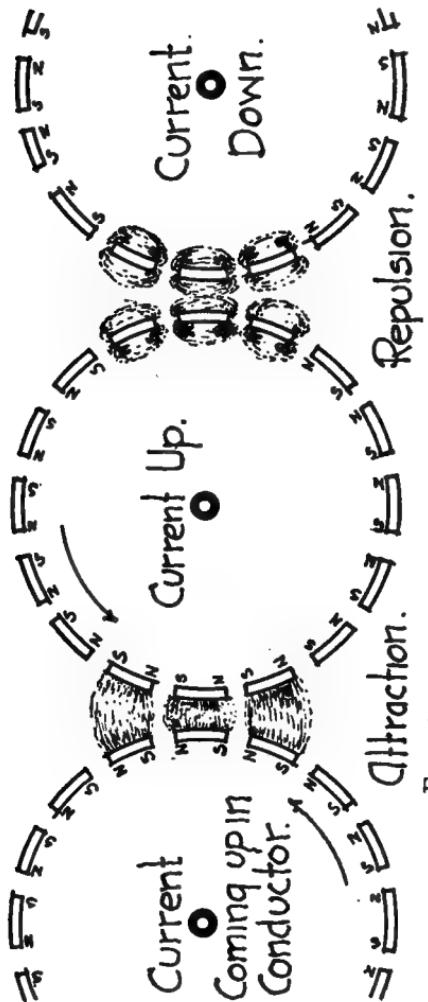
Fig. 15.

Fig. 16.

CHAPTER III.

In continuing our study of attraction and repulsion, we will consider the action of two magnet poles situated at a distance from each other. Let d represent the distance between them and m_1, m_2 their respective strengths. The attraction or repulsion between them would be $\frac{m_1 \, m_2}{d^2}$, or in other words the product of their strengths divided by the square of the distance. In proof of this we will substitute 1_1, 1_2 as the strengths and 1 centimeter as the distance and we find $\frac{1_1 \, 1_2}{1^2} = 1$ dyne of force, and we know this is true from the fact that unit pole at unit distance is attracted or repelled with a force of **one dyne** by another unit pole.

By increasing the distance to two centimeters we have $\frac{1_1 \, 1_2}{2^2} = \frac{1}{4}$ dyne, which conforms to the statement that the field strength varies inversely as the square of the distance.

If one of the poles is **doubled** in strength the force is doubled, and if both poles are doubled so that m_1, $m_2 = 2_1$, 2_2 the result of $\frac{2_1 \, 2_2}{2^2} = 1$ dyne, or 4 times as great a force as that exerted by the poles when their strengths were 1_1 and 1_2.

As shown in the last article **conductors** carrying current are attracted or repelled by each other, depend-

ing upon the **direction** of the flow of current. The force they exert upon each other is governed by the **length** of the conductor, the **amount** of current flowing and the **distance** between them. It can be expressed in the terms $\dfrac{2\,I_1\,I_2\,l}{d}$ = force in dynes, provided that the distance between them d, and the length of the conductor l, is expressed in centimeters and the current in absolute units.

As an example, we wish to determine the force exerted by two conductors upon each other when they are twenty feet long and five inches apart, with 100 amperes flowing through them.

As I_1, I_2 represent the current in absolute units, 100 amperes = 10 absolute units; therefore, I_1 and I_2 = 10 each.

As one foot = 30.48 centimeters, 20 ft. = 20 × 30.48 c. m. = 609.6 c. m.

5 inches = 12.7 c. m.

Substituting, we have

$$\frac{2 \times 10_1 \times 10_2 \times 609.6}{12.7} = 9600 \text{ dynes.}$$

As it is generally desired to get the result in **pounds,** the force in dynes must be divided by the number of dynes that one pound represents; and, as one pound = 453.5 grams and one gram = 981.45 dynes, one pound = 453.5 × 981.45 dynes = 445,087.575 dynes; therefore, the force of attraction or repulsion between these two conductors is 9600 ÷ 445,087.575 = .0216 lb.

If a compass is placed in a magnetic field, it will whirl into a position almost **parallel** to the lines of force. This action depends not only upon the strength of the poles of the compass and the strength of the field of force, but also upon the **length** of the compass needle.

This applies to bar magnets also. The product of the length by its pole strength is termed its **magnetic moment,** the symbol being M.

A bar magnet having a unit pole strength and unit length is said to have a **unit magnetic moment.**

As the earth's magnetism tends to hold the compass needle in a position north and south, a magnetic field whose lines flow in a different direction must overcome the force exerted by the earth's magnetism, thus producing a **deflection** of the needle. Therefore the compass is very useful in exploring the field surrounding a conductor. A long helix of wire exhibits the same characteristics as a compass or a magnet when carrying current.

The self-taught scientist William Sturgeon (1783-1850) made the first electromagnet worthy of the name.

It consisted of an iron core wound with eighteen turns of bare wire.

Instead of **insulating** the wire, he coated the core with **varnish** which accomplished the same result. With this crude arrangement he was able to lift a weight of nine pounds, which was about twenty times as much as it weighed. When the current was stopped, the magnet was unable to support the weight because the iron being soft had very little residual magnetism. A still more powerful magnet was constructed by Professor Moll of Utrecht. It was able to sustain a weight of 154 pounds.

These electromagnets were only laboratory toys and decidedly impractical.

In 1829 our own Joseph Henry made a step in advance of his contemporaries by improving the style of winding electromagnets, and **insulating** the wire with silk. Instead of winding the core with a few turns of large wire he used comparatively **small** wire with a **great number** of turns.

Professor Henry (1799-1878), of Albany, New York, was a teacher in a college at the time of his experiments and had no time for such work during the school term, but devoted himself to the study of electromagnetic phenomena during the vacation periods. He received his love for science by reading Gregory's " Lectures on Experimental Philosophy."

Professor Henry constructed his magnets with several magnetizing coils of wire with free ends which could be connected in a variety of ways. If the coils are connected so that the current in attempting to pass through one coil must pass through all the other coils consecutively, they are said to be connected in series, as shown in Fig. 17.

If each coil has the same resistance, it requires six times as great an e. m. f. to force as much current through them all as through one coil, which is, according to Ohm's law, $I = \dfrac{E}{R}$.

When connected in **parallel,** as shown in Fig. 18, it requires six times as much **current,** the magnet being the same strength as before, but it only requires as great e. m. f. as for **one** coil.

Because of these facts, the batteries should be connected in **series** to give the proper e. m. f. in the first instance, and in **parallel** to give the proper current in the second instance. Professor Henry observed these relations of e. m. f. current and resistance, in his " quantity magnet " as he called it.

He was unaware of Ohm's discovery in 1826 of these same principles, which principles underlie almost every branch of electricity, until told to him in 1837 while he was in London.

During his vacation in August, 1830, he noticed that an **induced** current was produced in a coil of wire wound on a piece of soft iron placed across the poles

4

B.
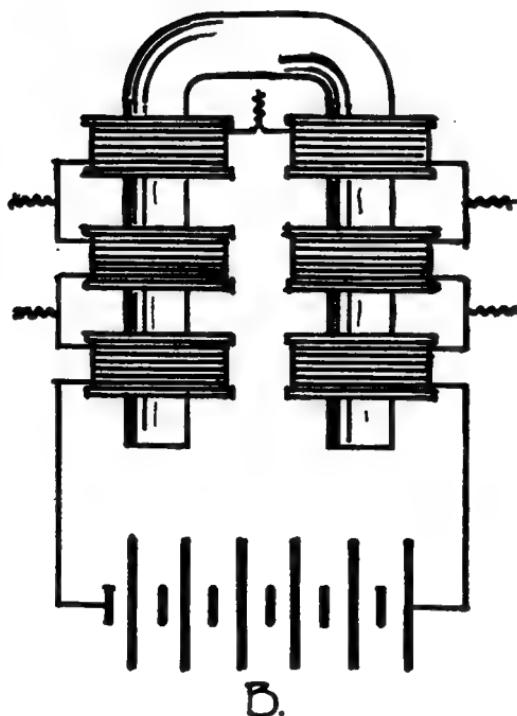
FIG. 17.
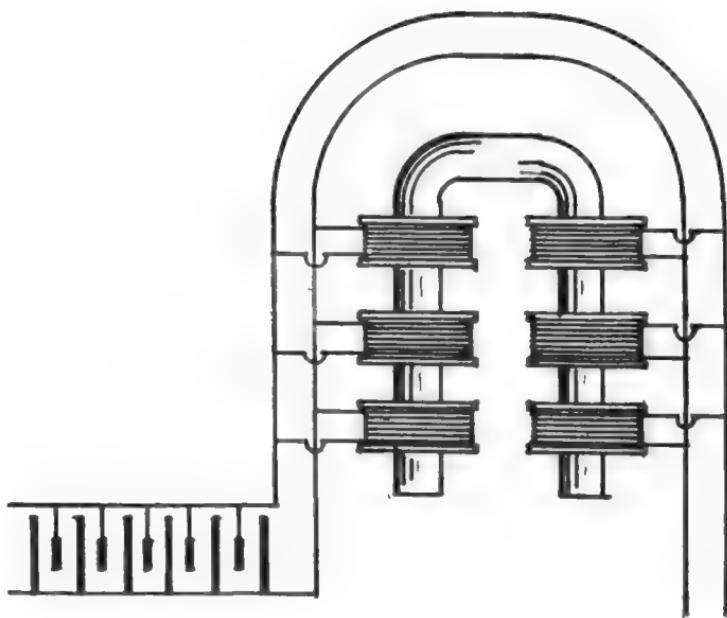
FIG. 18.
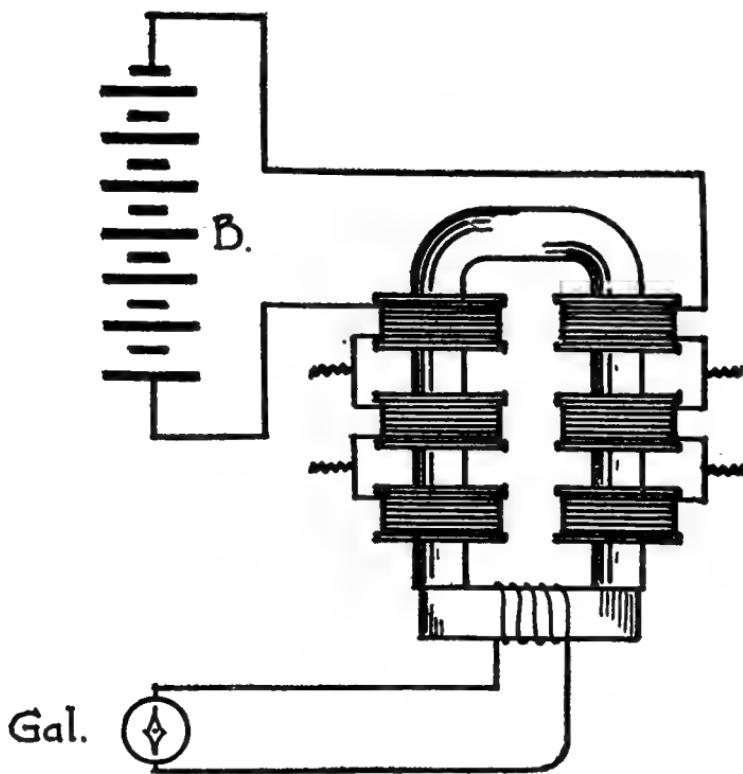
FIG. 19.

of his quantity magnet when the circuit of the magnet was broken. His method is shown in Fig. 19.

This was a remarkable discovery, upon which is based the principles of our gigantic **transformers** of the present day.

It will be understood from the following how a current can be produced in a coil which has **no electrical connection** with the source of electricity.

In Fig 20 is shown the magnetic field between two magnet poles marked north and south respectively. The lines of force are assumed to issue from the pole marked N and enter the pole marked S. When a conductor is moved through this field of force in the direction of the horizontal arrow, electricity will be generated which flows through the conductor in the direction of the vertical arrow.

When the conductor is moved in the **opposite** direction, the electricity flows in the reverse direction from that indicated by the arrow in the first instance.

When the upper magnet is changed to a north polarity and the lower one to a south, the current would flow **upwards** in the conductor, and not **downwards,** as shown in Fig. 20.

If the conductor remained **stationary** and the **magnetic field** moved across the conductor so that the apparent motion of the conductor would remain unchanged, the current would continue to flow in its original direction.

Of course, no current flows if the free ends of the conductor are not **connected** in some way, but an e. m. f. is generated, that is, a tendency to force a current in the direction stated as soon as the circuit is **completed.** The end of the conductor at which the current tends to leave, is termed the **positive** terminal; and is indicated by the plus sign, thus $+$; the other end is termed **negative,** and is indicated by the minus sign, thus —.
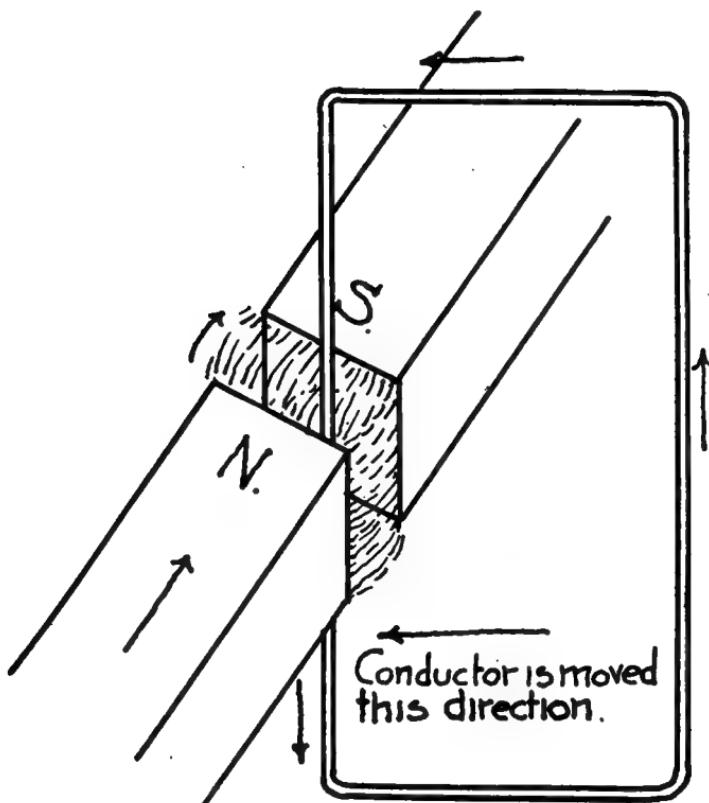
FIG. 20.

Prof. J. A. Fleming originated a very easy and convenient method to determine the **direction** of an induced current, if the direction of motion and the polarity of the magnet poles are known; or, either of the latter, if any two of the factors are known.

The forefinger in Fig. 21 represents the direction of the magnetic flux, the middle finger the induced e. m. f., and the thumb the direction of the motion of the conductor. When a conductor is moved, as shown in Fig. 20, it is said to be **cutting lines of force,** it being immaterial whether the conductor or the field is moved.

When the induced current flows, it produces a magnetic field around the conductor, as shown in Fig. 21. This field causes the conductor to be **repelled** by the
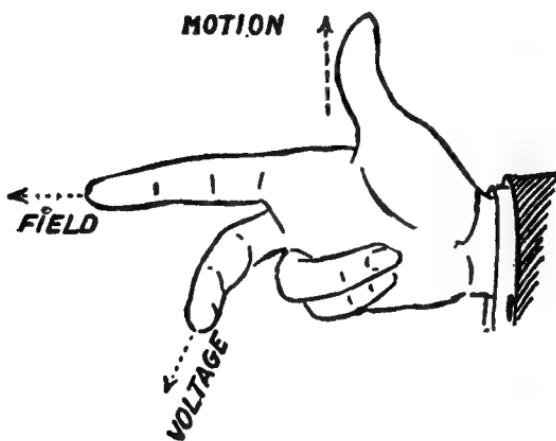
FIG. 21.

stationary magnetic field, for the reason given for the repulsion of two conductors carrying current in the same direction, as explained and illustrated in Fig. 16.

The lines of force (Fig. 22) on one side of the conductor marked A_2 are parallel to the lines A_1 issuing from the magnet pole N and as they are flowing in the **same** direction, they **repel** each other, and, therefore,
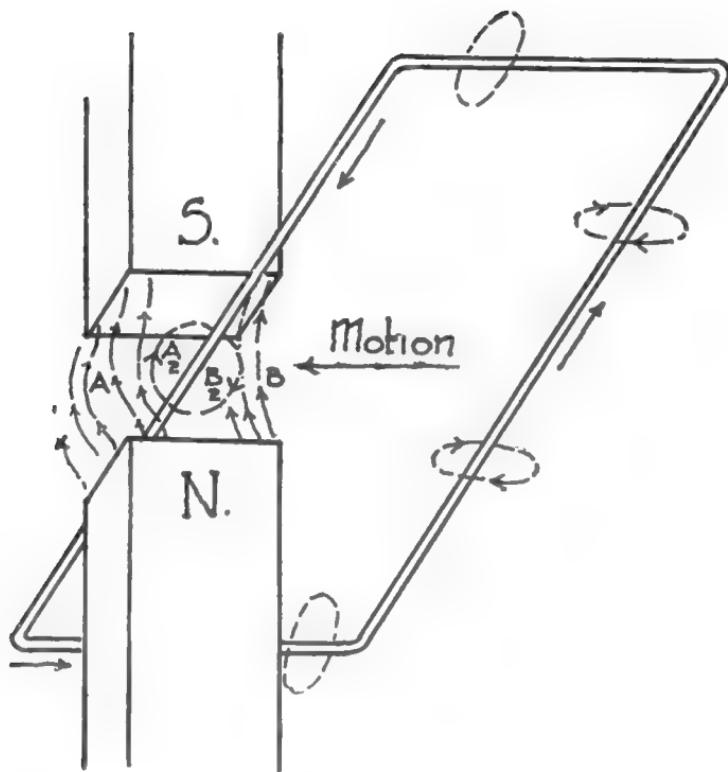
FIG. 22.

result in **opposing** the motion of the conductor. On the other side of the conductor, the lines B_2 and B_1 are also parallel but are flowing in **opposite** directions, which causes them to be **attracted,** and this opposes the movement of the conductor still more.

More current flowing through the conductor produces a stronger magnetic field, which in turn opposes the motion of the conductor with greater force.

Thus, it requires more work to move it in overcoming the **reaction** as the current is increased, and the value of the current is governed by the resistance of the circuit and the e. m. f.

The electromotive force depends upon the **rate** of cutting the lines of force.

When a conductor is moved **across** a unit magnetic field (which is described as containing in one line of force per square centimeter, $\mathbb{X} = 1$, being uniform throughout) in one second, thereby cutting one line per second, there is generated a **unit** e. m. f. If the conductor is moved twice as fast across the same field, the e. m. f. is doubled; but if the field is weakened the conductor must move faster to cut as many lines per second as before, and slower if the field is strengthened.

Summing up, we find the force opposing the movement of the conductor is governed by the strength of the field and the volume of current flowing, which is governed by the resistance of the electrical circuit and the generated electromotive force, which in turn depends upon the rate of cutting the lines of force.

If unit e. m. f. is generated in the conductor and we find that its motion is opposed with a force of one dyne, the conductor would be carrying unit current, and its resistance according to Ohm's law would also be unity, for $\frac{E}{I} = R.$ or $\frac{1 \text{ unit e. m. f.}}{1 \text{ unit current}} = 1$, or one unit resistance.

As unit e. m. f. is so exceedingly small, a **practical** unit, the **volt**, is used. It is equal to 100,000,000 times the **theoretical** unit, or it may be expressed as the e. m. f. generated by cutting lines of force at a rate of 100,000,000 per second.

Other forms of Ohm's law are as follows:

amperes $= \dfrac{\text{volts}}{\text{ohms}}$, or ohms $= \dfrac{\text{volts}}{\text{amperes}}$, or volts $=$

ohms $\times$ amperes. These are all **practical** units.

As the ampere is only .1 of unit current, and the volt

is 100,000,000 times unit e. m. f. the ohm $= \dfrac{100,000,000}{.1}$

$= 1,000,000,000$ times unit resistance, so unit resistance can be expressed in decimals as .000000001 ohm.

Unit work is done in unit time when unit **force** is exerted against the motion of a conductor through a unit field at unit speed; thus producing a unit current at unit e. m. f. through unit resistance.

The **work** done in this case is one **erg** per second, or one dyne overcome through one centimeter.

If the conductor in the illustration used is moved twice as fast per second, two units of e. m. f. would be generated, which would force two units of current through the electric circuit; therefore, the force is doubled, or is two dynes. As the conductor was moved two centimeters in one second, the force of two dynes was overcome through two centimeters; and, therefore, the work done is four ergs per second.

As the force in dynes opposing the movement of the conductor increases directly with every additional unit of current, and every additional unit of e. m. f. represents an increase in movement of one centimeter per second in a unit field, the work done must equal their product. As one **volt** represents a movement ·through a unit field of 100,000,000 centimeters in one

second to cut 100,000,000 lines of force, and as one ampere would exert only one-tenth of a dyne of force against the movement, the work done by one ampere flowing through a circuit at one volt would by $\frac{1}{10}$ dyne overcome through 100,000,000 centimeters, or 10,000,-000 **ergs** per second. This product is termed one **watt,** whose value is expressed volts $\times$ amperes $=$ watts.

As we found that 7,460,000,000 ergs per second equal one horse-power (see page 8, Chapter I) one watt equals 10,000,000 $\div$ 7,460,000,000 $= \frac{1}{746}$ **h. p.,** or 746 watts $=$ one horse-power.

When a current flows through a conductor a magnetic field is established around the conductor, whose value is changed when the value of the current is changed.

In the preceding paragraphs we considered the **conductor** as moving across the lines of force, but we will now consider the **lines of force** moving across a conductor.

As has been mentioned before, the passage of an electric current through a conductor produces a displacement of the omnipresent ether, and the lines of force thus produced are similar to an infinite number of consecutive rubber bands around the conductor.

This displacement is increased when the value of the current is increased, and is correspondingly diminished as the value of the current decreases.

When the current is **increasing,** the displacement represents a movement of the lines of force **outward** from the conductor. These lines of force return **toward** the conductor as the current is **decreasing,** the movement in both cases being at **right angles** to the conductor.

By placing another conductor, S, in the same plane

Lines are Expanding in this Direction

Conductor open

Current Increasing
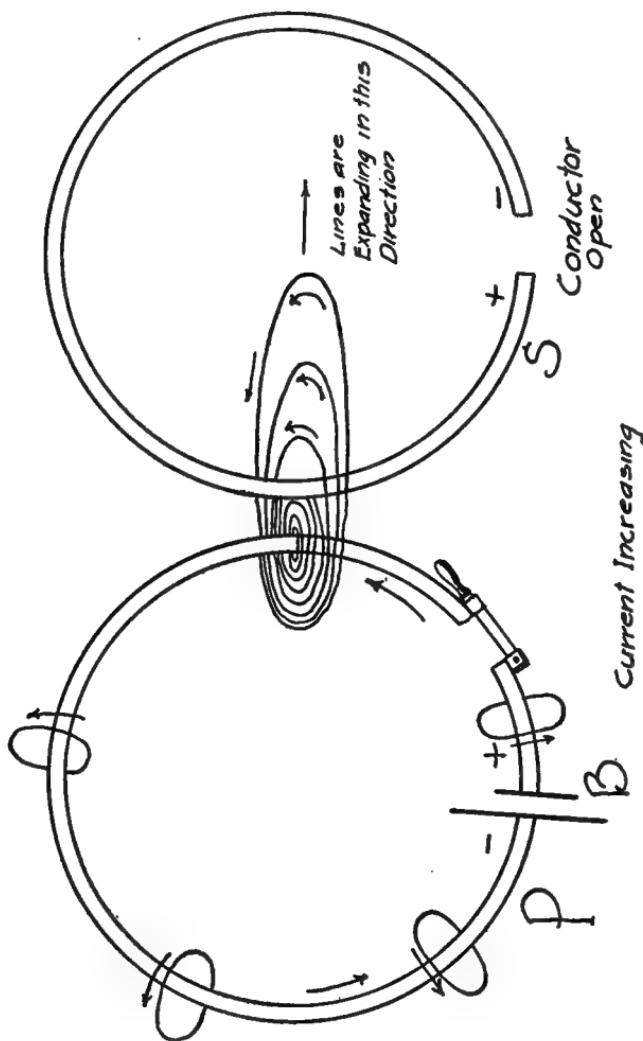
Fig. 23.

as the first, the lines of force must pass through it during their expansion, as shown in Fig. 23.

The letter P represents a conductor bent into a circle or turn, which is connected in series with a battery, B, or other source of direct current, and also a switch.

When the switch is closed the current rushes through the conductor, and in so doing produces a magnetic field around it. As the current does not attain its maximum value instantaneously, but requires an appreciable length of time, it is evident that the displacement of the ether surrounding the conductor must begin when the first infinitely fine current completes the path around the electrical circuit.

This displacement gradually becomes greater as the value of the current increases, forming ever-enlarging circles, which in their expansion are cut by conductor S (Fig. 23). Thus an e. m. f. is generated whose polarity can be determined by means of the Fleming three-finger rule. It should be observed that the virtual direction of motion of the conductor S is the reverse of that of the expanding magnetic field, while, in fact, the conductor does not move at all, being the converse of Fig. 20.

In Fig. 23 only electrical pressure is generated as the circuit is open, but when it is closed, as in Fig. 24, the circuit is completed and the e. m. f. forces current through it in the direction of the small arrows. This current is called the secondary current, because it is produced in a second conductor which is called the secondary coil, or simply the secondary.

The other conductor is called the primary, and the current is called the primary current.

The secondary current has exactly the same properties of producing magnetic fields as the primary, but it will be noticed that the secondary current is flowing in a direction opposite to that in the primary,

Expansion of Lines in this Direction

Current

S Conductor Closed

Current Increasing

B

P
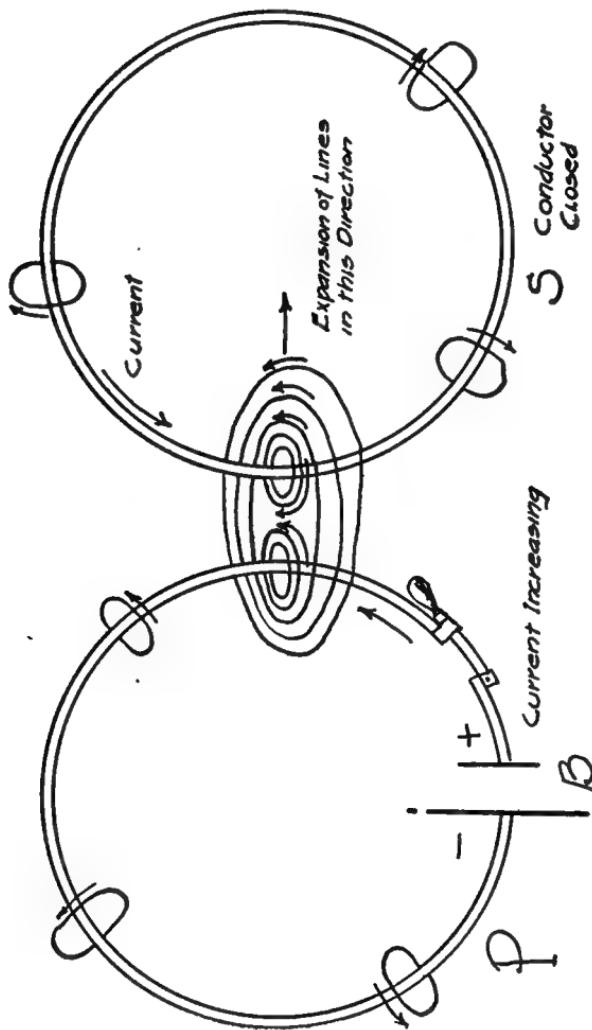
Fig. 24

thus producing a magnetic field whose lines of force rotate in an opposite direction to those of the primary.

In Fig. 25 it will be observed that the switch has been opened, thereby stopping the flow of current in the primary circuit. The result is that the lines of force which were expanding, being relieved of the magnetomotive force, collapse upon the primary conductor; and, in their return from an infinite distance, pass through the secondary. This again causes an e. m. f. and a current to be generated; but, in the opposite direction from the first, as the movement of collapse is the reverse from that of the expansion, although the lines of force continue to rotate in the same direction until the last trace of current has disappeared from the primary circuit.

One may wonder why the lines of force, in being cut by the near half of the secondary, should be taken as those which generate the e. m. f., when the lines in their expansion, or collapse, also are cut by the far one-half of the turn. The reason is that less lines of force cut the latter, in their expansion or collapse.

Let us consider the magnetic field produced around a conductor carrying ten amperes of current, and it may appear clearer.

In Fig. 26 the value of $\mathcal{H}$ is represented by the vertical line.

As currents flowing in the same direction are attracted by each other, the current flowing in a conductor can be considered as being composed of a great number of fine currents flowing in parallel. From this point of view it is seen that the greatest density would be at the center of the conductor. This is true only when the current is absolutely constant in value. The conductor is shown at the lower left-hand corner, and the distance at which $\mathcal{H}$ is to be found is represented by the horizontal line, which is divided into centimeters.

Lines of Force are Collapsing in This Direction

Current

S

Current Stopped

Fig. 25.

+

−

B

P
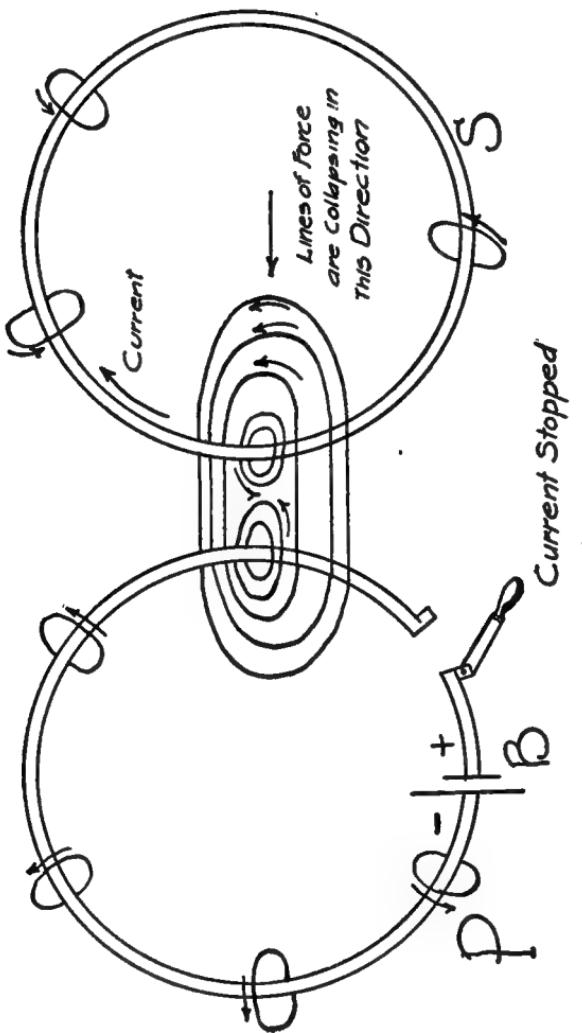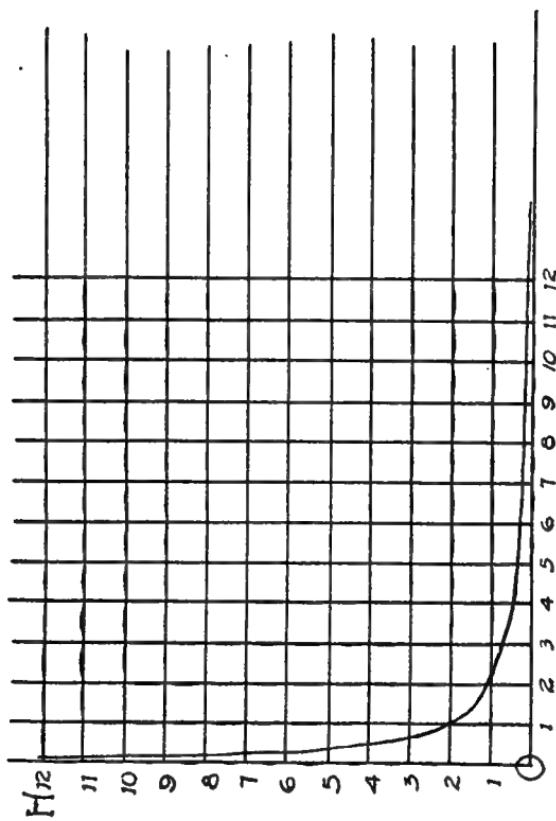
FIG. 26.

If we wished to know the **density** at a distance of 2 centimeters from the center of the conductor, find the figure 2 on the lowest horizontal line and see at what point in ✂ it coincides with the curve of flux density. We find that ✂ equals one line of force at this point, and if the value of ✂ with one ampere is desired, it can be obtained by dividing by 10, for

$$\maltese = \frac{.2 \times \text{current in amperes}}{\text{Distance in c. m.}}$$

At 1 centimeter from the center of a straight conductor, P (Fig. 27), carrying a current of ten amperes, the flux intensity ✂ $= 2$ when the value of the current is constant. When another conductor S_1 is placed parallel to conductor P, and at a distance of 1 centimeter from it, it lies in a magnetic field whose intensity is ✂ $= 2$. S_2 in Fig. 27 is the return conductor of S_1, forming a closed circuit, and lies at a distance of 10 centimeters from P, therefore, lying in a magnetic

field whose intensity is ✂ $= \dfrac{.2 \times 10}{10 \text{ c. m.}} = .2$ line.

These conductors are assumed to be very long and straight, or this would not be true. When **no current** flows in the primary, of course no magnetic field is produced; so it is evident that all the lines of force, from the line marked f_1 to infinity, must have passed through conductor S_1, and that all the lines from f_2 to infinity must have been cut by S_2, from the time the current in P was zero until it reached a value of ten amperes.

As both S_1 and S_2 cut the expanding (or contracting) lines in the same manner, the result is that the e. m. f. generated by each is of the same sign.

If they were of the same value, of course no current would flow, because they would exactly balance each other; but this is not the case, as S_1 cuts more lines in the same time than S_2.
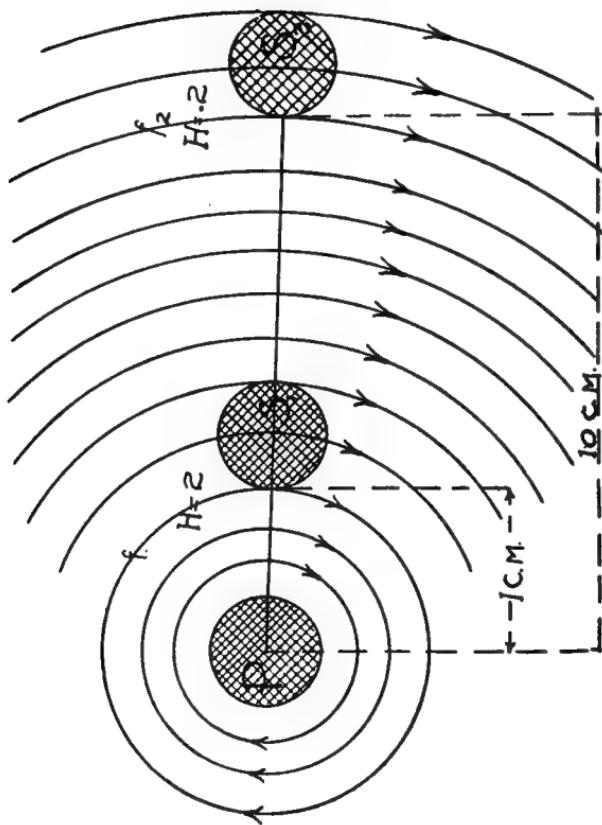
FIG. 27.

All the lines of force between f_1 and f_2 have passed S_1, but not S_2, therefore, the e. m. f. of S_1 exceeds that of S_2 by the e. m. f. generated by the lines between them.

The flux between P and S_1 does not pass S_1, so it produces no inductive effect in the secondary. It is called the **magnetic leakage.** As only a fraction of the lines produced in Fig. 23 **thread** the secondary coil S, there is said to be a great magnetic leakage, and they are very **loosely coupled.** By placing the secondary in the position shown in Fig. 28, this leakage is greatly
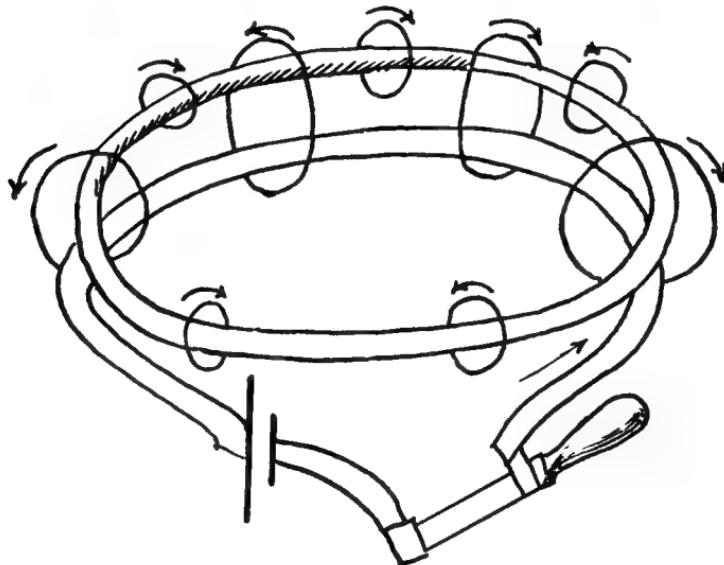
FIG. 28.

reduced, as most of the lines are made to thread the secondary. Such a transformer, as described, is called an **air-core** transformer, because **air** is the magnetic

medium. By substituting **iron** for the air, the induct-ive effect is greatly enhanced, as it is then possible to produce a greater flux.

As the magnetic flux produces no induced **current** in a conductor lying within it, except the lines **are cut** by it, and as the rate of change should be **kept up** constantly, it is evident that some other way than such as a switch worked by hand would be necessary.

To accomplish this, electric generators have been constructed to deliver an ever-varying current, **which** is said to be **alternating.** It is abbreviated A. C. An alternating current is defined as one which is increased from a zero value of a positive sign, to a maximum value and then to zero again, only to be **reversed** to the negative sign, from zero to maximum and then to zero again.

A complete change, from zero in one direction through **all** the changes mentioned to zero again, is termed one **cycle.** As the alternations are produced many times a second, the number of cycles occurring in that time is called the **frequency.**

The frequency of commercial generators is sel-dom over a hundred per second, and generally less, the most common being at sixty cycles, although twenty-five cycles are quite commonly used.

CHAPTER IV.

As we enter the field of **alternate currents** our subject becomes more interesting and even fascinating, for in it there is much that is strange. Its usefulness **commercially** is constantly increasing, and many new applications are being found for it.

An embryonic type of generator for producing alternating currents is shown in Fig. 29. Its design is similar to the familiar magneto with its permanent magnets.

The upper arrow indicates the lines of force which are assumed to issue from north pole to the south pole of the magnet S, N.

If both sides of the coil a, b, c, were moved upward (or downward) through the field, it is evident that no current would be perceptible, as the sides a, b and a, c would be cutting lines of force in the same manner; therefore the induced e. m. f. at the collector rings R_1 and R_2 would be of the same sign and value, the result being that their respective e. m. f.'s exactly **neutralize** each other. If, however, the coil is rotated in the direction of the arrow at the collector rings, the side a, b, will be moving downward while the side a, c, will be moving upward correspondingly. In this way it is apparent that the induced e. m. f. at R_1 and R_2 would be opposite, positive at R_1 and negative at R_2, at that point of revolution, thus producing a tendency of the current to flow, if a conductor connects the rings.

As the induced current produces a magnetic field around the conductor which opposes its motion, it is plain that work must be done to move it through the
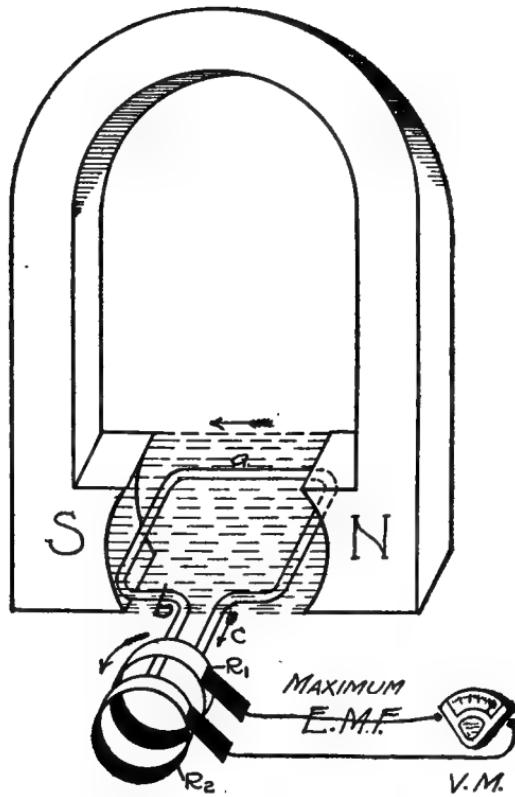
FIG. 29.
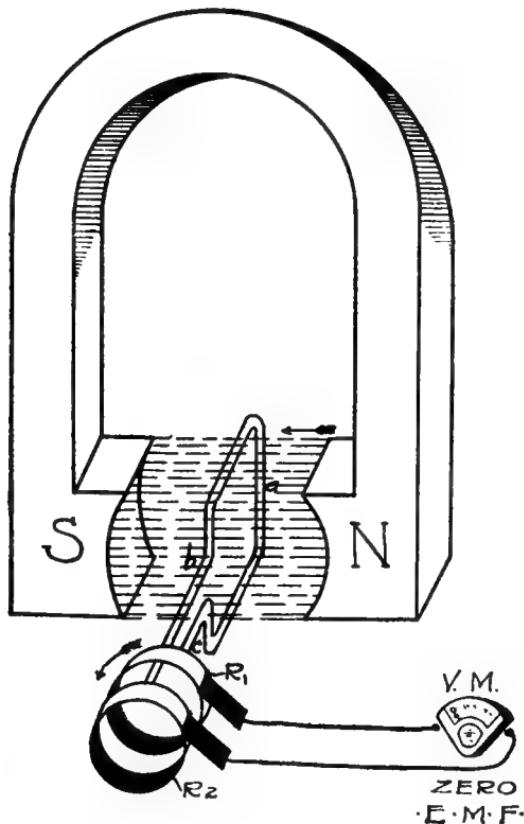
FIG. 30.
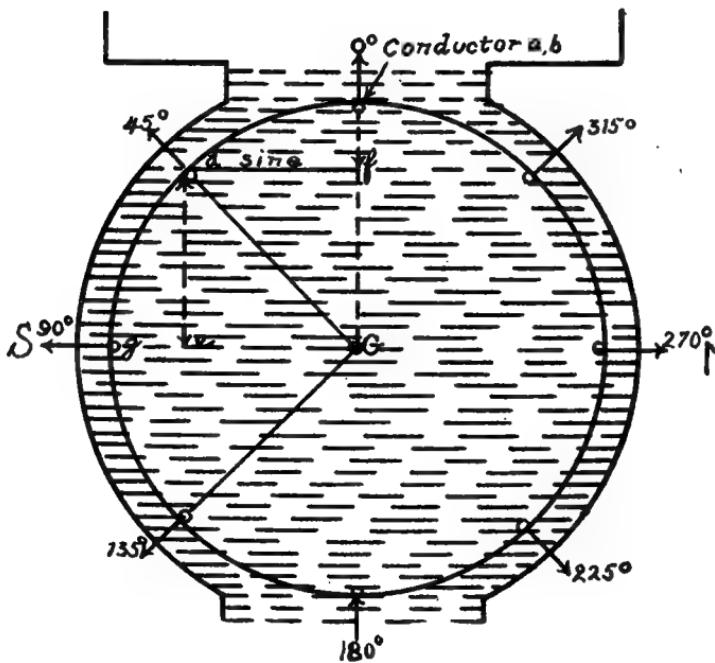
FIG. 31.

field, and as the resistance of the external circuit is decreased and more current flows through the coil a, b, c, it reacts more strongly in opposing the motion, therefore requiring more power to turn it.

By carefully observing the coil in Fig. 30 during one revolution in the direction of the arrow near the collector rings, it will be noticed that in its vertical position the coil is not cutting any lines but is moving in a direction parallel to them. This shows that at that point of the revolution no electromotive force is generated, in other words, its value is zero. When the conductor a, b, is followed through one complete revolution it will be seen that it does not cut lines of force at the same rate at every point of its revolution. This is made plainer by consulting Fig. 31. As the conductor a, b, travels in a circle, every revolution can be considered as passing through 360° of position, as every circle is assumed to have 360°.

The conductor in its movement from the position 0° to 45° does not move across the lines very fast, as they are almost parallel to its movement.

The number of lines that it cuts in that distance is the number embraced by the line d, f, and the conductor at 0°.

In advancing from the position 45° to 90° the conductor cuts the lines more rapidly as it moves across them nearly at right angles. It can readily be seen that more lines are embraced between the points d and e than between the lines d, f, and a, b, although the distance the conductor advanced was the same in both cases, namely, ⅛ of a revolution.

As the induced e. m. f. depends upon the rate of change, it is obvious that, as the lines are being cut more rapidly from 45° to 90° than from 0° to 45°, the e. m. f. is higher. At the position 90° the lines are at right angles, thus producing at that point the highest e. m. f. From this point to 180° the rate decreases to

zero and from 180° to 360° (or 0°) the former opera-
tion takes place, only the e. m. f. is now of the **opposite**
sign, as the conductor is moving upward through the
field.

The different values of the e. m. f. for one revolu-
tion are shown graphically in Fig. 32.   The curve
above the line represents the e. m. f. of one sign, and
the curve **below** the line that of the other sign.   The
·**horizontal** line represents the advance in degrees and
the time, if the **frequency** is known.   It is termed a
sine wave for reasons that will be shown presently.
Each one-half of this cycle is termed an alternation.
All waves do not have the same **shape,** this depending
upon the shape and position of the poles and irregu-
larities in the density of the field, and other causes
that will be discussed later.

The **height** or amplitude of the wave gives the
maximum voltage generated, but this is not the
effectual voltage.

The effectual voltage means its apparent voltage
when applied to a resistance which is not changed by
the frequency, as an incandescent lamp.   When the
current and resistance are known, the **apparent** voltage
can be obtained by Ohm's law, $R \times I = E$, where R
is resistance in ohms, I is the current in amperes and
E is the e. m. f. in volts.   How to determine the **true**
maximum e. m. f. when the effectual voltage is known,
or vice versa, depends upon the **form** of the wave.

The number of lines cut by the conductor in Fig. 31
does not vary directly with its advance, but its result-
ant movement at right angles to the field.   This is
shown in Fig. 33.

The field in this case is assumed to have a flux of
10^8 mega-lines, or one billion lines of force.   If the
conductor c is moved to b in one second, it cuts 10^9
lines per second, thus producing an e. m. f. of 10 volts.
If the same conductor is moved from c to a in one
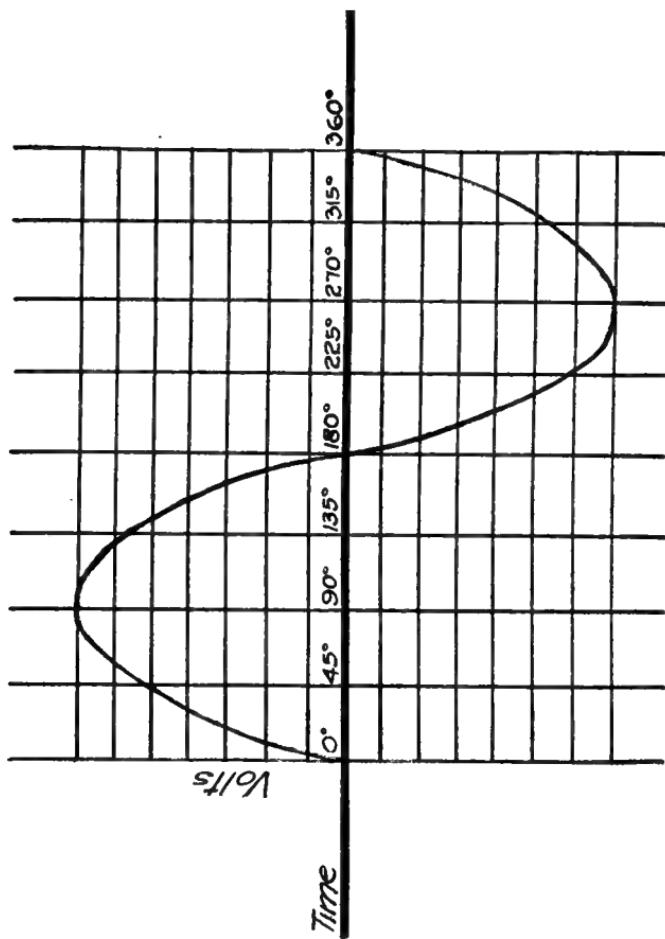
FIG. 32.
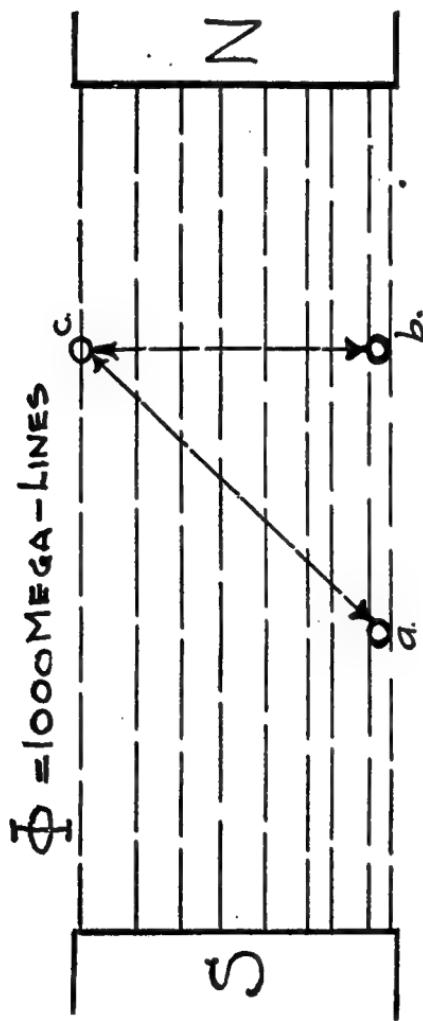
Fig. 33.

second, it has to travel faster to do so and yet it does not cut any more lines of force than it did in the shorter distance.

We see from this that the **number** of lines cut by a conductor depends upon its effectual movement at right angles to the lines, and **not** the arc of the circle it passes through.

It can be proved by trigonometry that the rate the conductor, a, b, Fig. 31, cuts the lines of force at any instant and consequently the generated e. m. f. varies directly with the line d, f. The line d, f gets longer as it moves from a, b to g, at which position it is longest and lies along the line g, c. In trigonometry the line d, f represents what is called the sine of the angle d, c, f.

For this reason the graphical representation of the values of e. m. f. or current is called a **sine wave.** If the pole-piece is so designed that the lines do not issue uniformly, it is clear that the cutting of the lines of force will not vary uniformly as a sine wave, but will have irregularities. If the pole-pieces have spots of iron of **low** permeability, the wave becomes irregular also.

As the reluctance of air is very great, the flux density of the field between the poles S and N in Fig. 29 can be greatly increased by winding the coil upon an **iron core** called the armature. This increases the induced e. m. f. in the coil, but by increasing the number of turns upon the armature, it can be further increased. If one of the collector rings is removed and the other is split into two halves, as in Fig. 35, and the ends of the coil are connected to each one-half of the ring respectively, a direct current can be obtained.

The current is then said to be a **rectified** current. That it is not very smooth is evident by the shape of the wave it produces in Fig. 36. If two coils are wound

S. N.

-R₁

R₂

V. M.

ZERO
E.M.F.
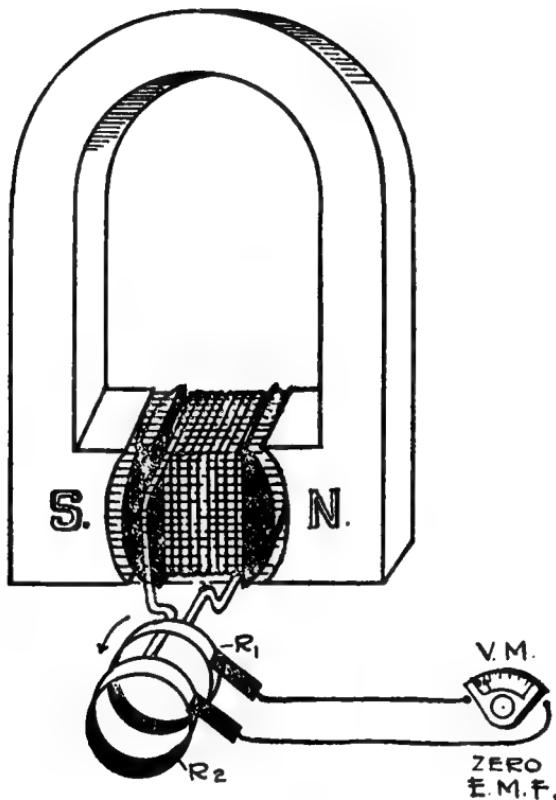
Fig. 34.
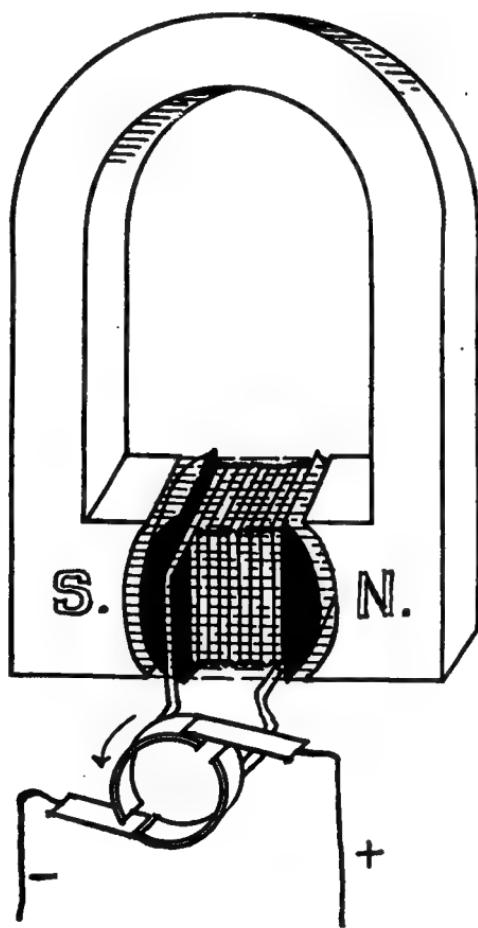
FIG. 35.
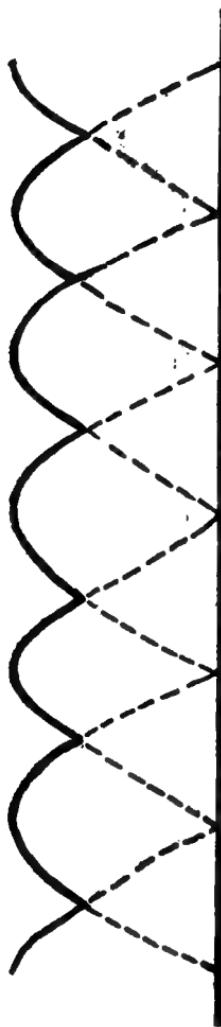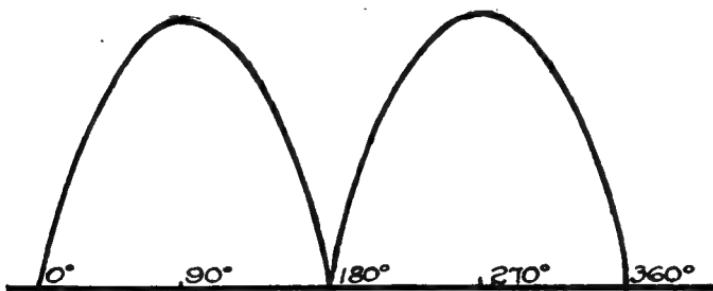
FIG. 37.

FIG. 36.

around the armature at right angles to one another and their four ends connected to a ring of four segments, the wave would present a form similar to Fig. 37. A ring made up of several segments used in rectifying the current in the armature, is called a **commutator.**

FIG. 38.

By winding a great number of coils on an armature, each in a different plane and connecting them to a great number of segments, a current can be obtained which is comparatively smooth, as shown in Fig. 38.

The wave is almost **flat** in this case, as only the **peaks** of the waves are commutated, the dotted lines showing the form of the wave in each armature coil.

This shows that the current in the **armature** of a commercial direct-current machine is alternating.

As the alternating current generator in Fig. 29 is

unsuited for use in many cases of commercial use, generators have been constructed with a great number of poles; and, instead of using permanent magnets, powerful **electromagnets** are used.

There are several serviceable types upon the market, namely, the revolving **armature,** the revolving **field,** and the inductor alternators and rotary converters.

The first type is essentially the type previously described, Fig. 29, but, with a multiplicity of poles, as in Fig. 39.

As the poles are electromagnets, it is necessary to have a small **direct**-current dynamo to furnish current to **excite** them. This is called an exciter.

The armature in Figs. 29, 30 and 31 makes but **one cycle** per revolution, but in a generator with a number of poles, several cycles may be produced per revolution. This will be understood by referring to Fig. 40, where the armature is represented by the dark portion, the field magnets by the light portions marked S_1, N_1, S_2, N_2 respectively, and the conductors or coil winding by T_1 and T_2. The arrow indicates the direction the armature is moving.

No e. m. f. is generated at the positions shown, but as the armature moves under the magnet poles the conductors begin cutting lines of force. When the conductors lying between S_1 and N_1 are moved in the direction of the arrow under the pole N_1, the terminal T_1 becomes negative — according to the rule of Fig. 21, reaching its highest value when directly under the pole N_1 and decreasing until exactly **half way** between N_1 and S_2. From this point and onward under the pole S_2 the terminal T_1 becomes positive, according to the rule before referred to, reaching its **highest** value when directly under the pole S_2 and then **decreasing** to zero again when half-way between S_2 and N_2.

By referring to the wave below the generator it will
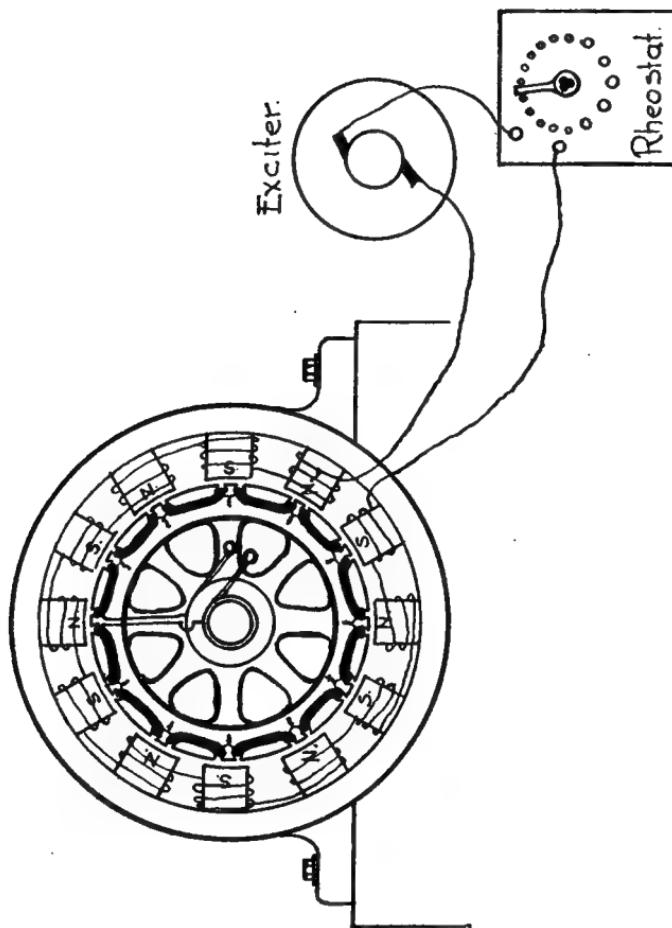
Fig. 39.
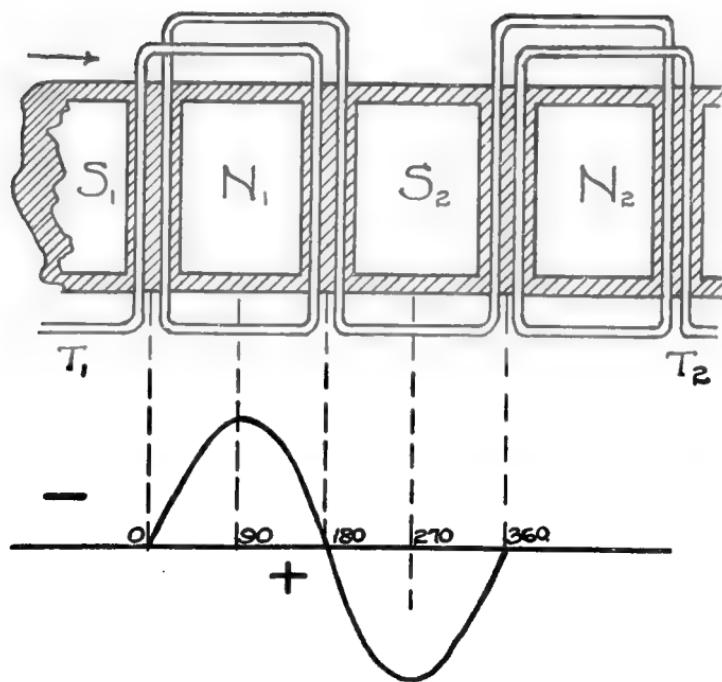
Fig. 40.

be observed that the e. m. f. passes through one cycle of values in only **a part** of a revolution. Because of this, the conductor is said to have passed through 360 magnetic degrees.

The number of cycles per revolution can be obtained by dividing the number of its poles by 2.

For this reason a **two-pole** machine would have to be run at the high speed of 3,600 revolutions per minute to obtain a frequency of sixty cycles per second.

The required **speed** of any generator when the number of poles are known, to obtain a desired frequency, is determined by the formula,

$$\text{r. p. m.} = \frac{2 \times 60 \times f}{\text{number of poles}}$$

where f is the desired frequency.

If the frequency is required when the r. p. m. and number of poles is known, the formula takes the form

$$f = \frac{\text{r. p. m.} \times \text{number of poles}}{2 \times 60}$$

The number of poles required for any speed or frequency could also be obtained in a similar manner if it were not for the fact that the answer must be a whole number divisible by 2 because a machine can not be constructed with an **odd** number of poles.

In Fig. 39 the rotating coils are connected in series and brought down to two collector rings on the shaft, where the current is taken from the machine by brushes pressing upon the rings.

The conductors are imbedded in the armature core which is made up from sheet steel. On large machines the sheets are not punched in circular discs with the slots for the conductors, but are made up from **many** small punchings firmly fastened in place upon a large cast-iron spider. The reason for making the armature from **sheet metal** will be explained subsequently.

In Fig. 41 is shown a part of the revolving field type of alternators which has several distinct advantages over the revolving armature type. First, the current is generated in a stationary armature which
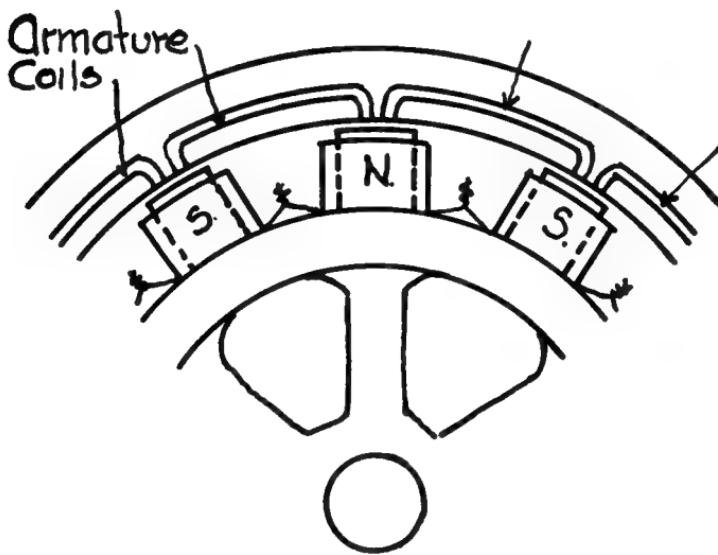
Fig. 41.

permits of better insulation; and, secondly, this obviates the necessity of **commutating** a very heavy current; and, thirdly, gives better opportunity for cooling.

Generally **both** the armature and field poles are laminated, that is, composed of sheet metal.

The inductor type of alternators is unique in that the only moving part is the **field core,** as shown in Fig. 42.
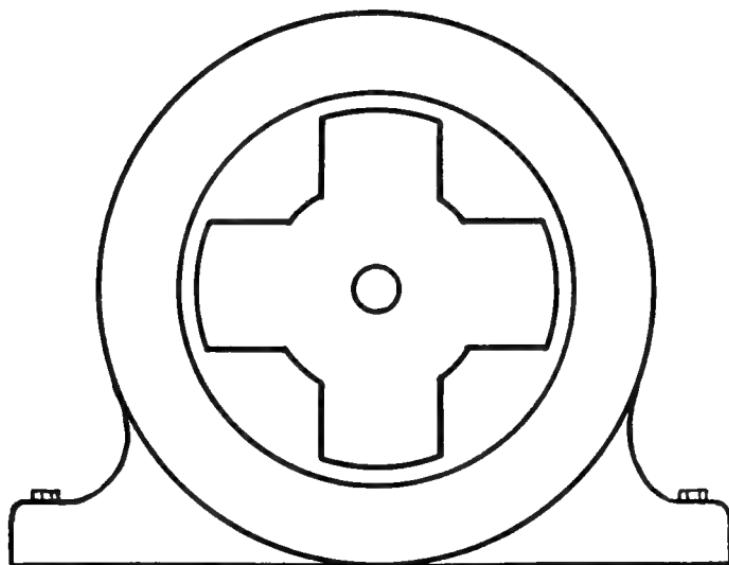
FIG. 42.

The exciting coil T_1 and T_2 in Fig. 43 is wound on a stationary frame around the middle portion of the rotating field.

Fig. 44 shows the plan of generation.

Pulley to run Exciter

Armature Coils

S S S

N N N

T₂

T₁

To Exciter
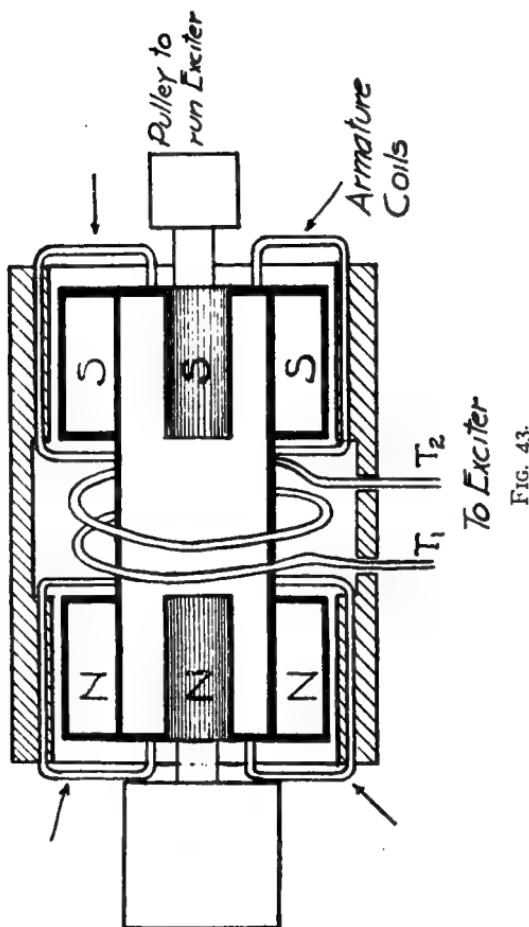
Fig. 43.
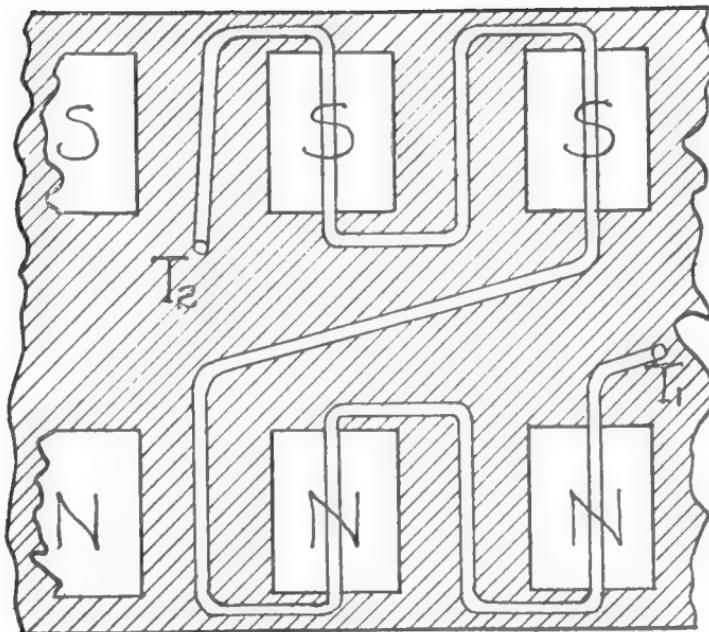
FIG. 44.

CHAPTER V.

When alternating current is required and the only available source of supply is direct, some means must be devised to efficiently change it to alternating; or, when the available current is alternating but of a different **frequency** from that required, it also becomes necessary to provide some means to obtain the desired frequency.

In case the current required is direct and the supply is alternating, or direct current of too high or too low an e. m. f., it again becomes necessary to devise some means to accomplish the conversion of the supply to that desired.

Means have been devised to produce the aforementioned results for their respective requirements, through the medium of machines termed **motor-generators.** They are also sometimes called **rotary-converters** or **rotary-transformers.**

A **motor-generator** which is supplied with alternating current at one frequency and delivers a current at a different frequency is termed a **frequency-changer.**

As their name implies, motor-generators consist properly of two parts, namely a motor and a generator, which parts are combined in one machine.

In some types, these machines consist of a motor designed to operate from current furnished by the source of supply, this motor being directly connected by its shaft to that of a generator which is designed to furnish the required quality of current.

When the source of supply is **alternating** as well as the generating end of the motor-generator, it becomes

necessary to use an **exciter,** that is, a small d. c.
dynamo to magnetize the fields of the generator,
which it is unable to properly do itself.

The exciter is sometimes directly connected to the
shaft of the motor-generating set, but more often it is
run by a belt from the motor-generator's shaft as the
exciter can be greatly reduced in size by running it at
a high speed. Such speed could not otherwise be
obtained, being **direct** connected, as the converters do
not generally run at a very high speed.

When the mains of supply are direct current it is
unnecessary to use an exciter, as this current can also
be used to excite the fields of the generating side.

It is not essential in all types of converters to have
two distinct machines, as they can sometimes be com-
bined more closely by using both the armature and
fields in common, and sometimes even the armature
winding is used in common by the motor and gener-
ating sides.

A **synchronous** motor-generator is one whose motor
is designed to run at a constant speed, determined by
the relation of the number of its poles to the frequency
of the alternating current that it receives at the mains.

Such a machine is not self-starting, so an **induction-
motor,** which is self-starting, is directly connected to
its shaft to bring it up to synchronism, which means
that its armature coils pass its field poles at the same
rate as the frequency of the current at the mains.

In case it is desired to obtain alternating current,
when in possession of a bipolar d. c. generator, it can
be accomplished by tapping the armature winding at
two opposite segments of the commutator, and leading
these taps to two collector rings firmly fastened to
and insulated from the armature shaft.

If there is no room on the shaft between its bear-
ings, the shaft can be bored lengthwise to the point
where tapped, and the leads brought out through this

hole to collector rings situated on the other side of the bearing.

Of course, a four-pole d. c. generator would be better suited for this than a bipolar, as the bipolar machine would have to be run at a speed of 3,600 r. p. m. to give even the common frequency of sixty cycles per second, while the four-pole machine would only have to run at one-half that speed to obtain this frequency.

When a four-pole d. c. generator is used which has four sets of brushes at the commutator, two opposite segments are connected together to one of the collector rings, while two other segments in quadrature to the first two are connected together to the other collector ring. These machines can be connected to direct current and run as motor-generators, or run by a belt from a source of power. As motor-generators they are not as satisfactory as when the motor and generator parts are distinct and connected by a shaft, as in the first case the current delivered at the collector rings robs the armature of a part of its current and as a result the armature runs at an excessive speed when very much current is delivered.

Resuming our study of induction, we will consider some properties of an electric circuit. As mentioned before, it is quite a simple matter to obtain the resistance of direct-current circuit by a form of Ohm's law $R = \dfrac{E}{I}$ But when the resistance of an alternating circuit is to be determined, the problem becomes more complex.

The resistance of a direct-current circuit is governed by the **length** and **cross-section** of the conductor and the **material** of which it is composed, while in alternating circuits there are other factors which are sometimes more important in arriving at the resistance than the factors already named.

In addition to the resistance of the circuit depending upon its length, cross-section of conductor and the material composing it, the resistance of an alternating-current circuit is governed by its **shape, permeability** of medium surrounding it and the **amount** of that medium.

It has been previously shown in our study of induced currents that the lines of force surrounding the primary conductor set up an induced electromotive force in a neighboring conductor as these lines expand and collapse. We found that the induced e. m. f. in the secondary circuit was of the **opposite** sign from the impressed e. m. f. in the primary circuit when the lines of force were expanding, and of the **same** sign when they collapsed.

The induced e. m. f. not only manifests itself in a neighboring circuit but also reveals its presence in the primary circuit itself, because the lines of force in their inward or outward movement from the conductor cut across other parts of the same circuit.

When the induced e. m. f. takes place within the same circuit as the impressed e. m. f. it is termed the counter or back e. m. f. and is indicated by a small subscript c or b, as e. m. f. $_c$ or e. m. f. $_b$

The effect of the counter e. m. f. is to **oppose** the flow of current in a circuit. In this way it acts as a **resistance.**

The properties of a circuit to produce this counter e. m. f. is called its self-induction or **inductance,** and is represented by the letter L. To distinguish between the **natural** resistance of a circuit and the resistance due to self-induction, the latter is termed the **inductive-reactance** of the circuit.

Self-induction is a very important factor in the subject of alternating currents, and exceptionally so in those of high frequency, so it is well that the student masters this part of the subject.

As long as the value of the current in a circuit remains **unchanged,** the inductance of a circuit does not manifest itself, but as soon as the current is varied in strength the inductance opposes the change.

When the conductors are long and straight they are said to have distributed inductance, but when they are formed into coils or helices they are said to have a concentrated inductance.

The inductance of a long straight wire is small compared to its inductance when formed into a coil; the greater the number of its turns the larger they are, and the closer they are together the greater is its self-induction.   The reason for this is that the different parts of the circuit are placed in a position more favorable to mutual induction between those parts.

When the turns of a coil are placed close together there is less magnetic leakage between them.   In other words, more of the flux produced by the current in one turn interlinks with the others.   As there is mutual induction between turns only when they interlink, it is obvious that the mutual induction between turns is governed by the flux which interlinks them — the greater the flux, the greater the mutual induction.

But self-induction does not depend entirely upon the mutual induction between different turns, as a conductor of only **one turn** has self-induction; in fact, any conductor, however straight, has a certain amount of inductance.

A two-wire circuit can be considered as a coil of one turn with very long sides.   The existence of inductance in such circuits is due to the fact that the flux surrounding the conductor has cut through it as it expanded and also cuts it in its collapse, generating in itself a counter e. m. f.   Of course no current flows in the direction chosen by the counter e. m. f. when the lines expand, only a **diminishing** of the value of current possible to force through the circuit.   The **unit** of self-

induction is called the **henry** in honor of that great
scientist Joseph Henry, as previously mentioned.

By definition a circuit has a self-induction of **one
henry** when it generates a counter e. m. f. of one volt
when the current is varied at a uniform rate of one
ampere per second — that is, the circuit cuts 10^8 lines
per second; so a circuit of one turn, which has a flux of
one Weber (10^8) when one ampere is flowing through
it, has an inductance of one henry.   (See Fig. 45.)

ϕ = 100,000,000 Lines
with one ampere
of current in 1 turn
L = 1 Henry.
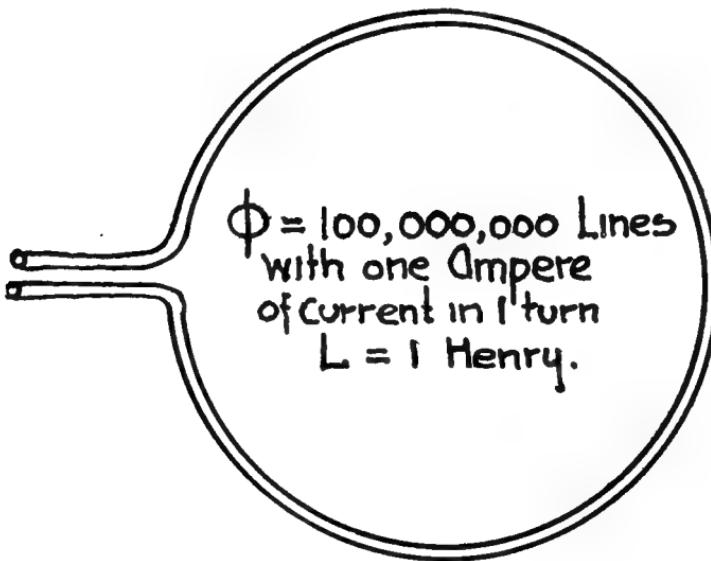
FIG. 45.

If the circuit described above, instead of having
one turn of conductor, is made with **two turns** of ex-
actly the same shape and size placed close together, as
shown in Fig 46, it is plain that the flux would be
doubled when an ampere current is flowing, as there

Φ = 200,000,000 Lines with one Ampere flowing in two turns L = 4 Henries.
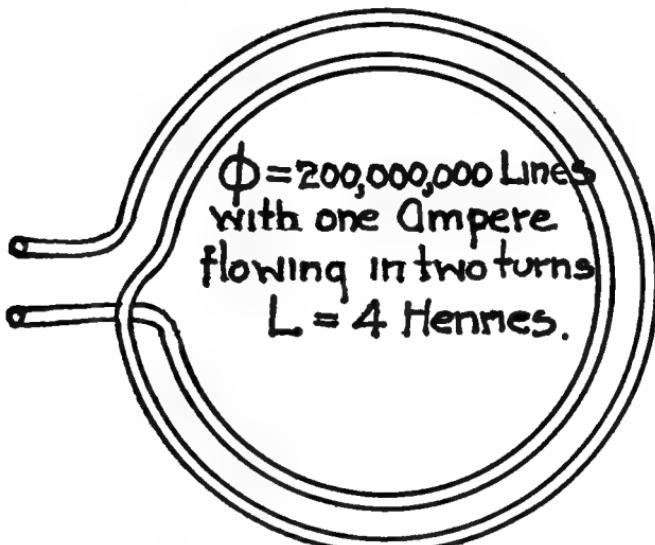
Fig. 46.

would be two ampere-turns instead of one. Doubling the flux would make $\Phi = 200,000,000$ lines of force.

This flux in its expansion, or collapse, upon one turn, would generate a counter e. m. f. of two volts, and as the two turns are in **series** the counter e. m. f. would be twice as much, or four volts.

According to the definition of a henry this circuit would have an inductance of four henries; therefore, the inductance increases as the **square** of the turns when the total flux interlinks them all. In this case $L = \dfrac{S\Phi}{I10^8}$ in henries, where S equals the number of turns.

The inductance of a circuit is sometimes expressed in centimeters, one of the c. g. s. units or absolute units.

It has been previously mentioned that one volt

7

represents a movement of a single conductor of 100,-000,000 centimeters per second across a unit field (1 line per sq. cm.). In this way the e. m. f. in volts can be given the dimension of **length,** namely 10^8 centimeters.

As a circuit of one henry inductance generates a counter e. m. f. of 100,000,000 centimeters when a change of one ampere, or one-tenth unit of current per second, it is plain that the counter e. m. f. would be ten times as high, or ten volts (1,000,000,000 cm.), when the rate of current change per second is unity (or ten amperes per second). Therefore, one henry of inductance is given the dimension of 1,000,000,000 centimeters. In a circuit of only one turn the inductance in centimeters can be directly obtained from the number of lines enclosed when a current of ten amperes is flowing through the conductor.

When the turns of a conductor are widely separated, the inductance in centimeters is equal to ten times the sum of the flux threading each individual turn, divided by the current.

$$L \text{ in cm.} = \frac{10\,(\Phi_1 + \Phi_2 + \Phi_3 + ---)}{I} \text{ where I}$$

is the current in amperes.

An example of this is shown in Fig. 47, where the letters S_1, S_2, S_3, S_4 and S_5 represent the individual turns of the conductor.

Because of the wide spacing of the turns, many lines produced by S_3 do not reach S_2 or S_4 and many of the lines produced by S_2 and S_4 do not interlink with S_1 and S_5.

Assuming that a current of 100 amperes traverses the circuit, and that 20,000 lines interlink the turn S_3, and 15,000 lines each through the turns S_2 and S_4, and 10,000 lines each through the end turns S_1 and S_4, we find by substituting these values in the formula
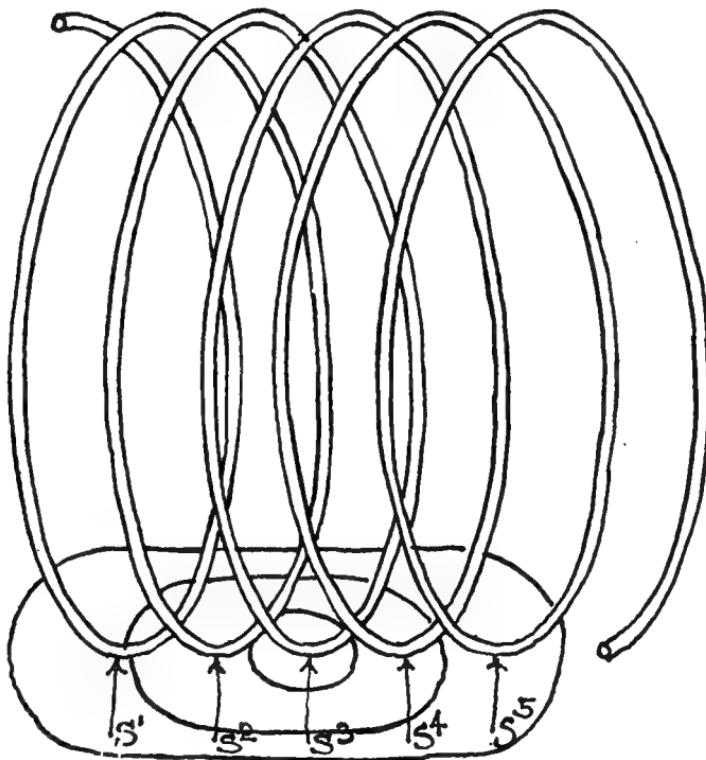
FIG. 47.

$$L = \frac{10(20,000 + 15,000 + 15,000 + 10,000 + 10,000)}{100}$$

$= 7,000$ centimeters.

If it is desired to convert the answer to henries it may be obtained by dividing that number by 10^9 or (1,000,000,000). In this case the answer in henries would be $\frac{7000}{10^9} = .000007$ henry.

As it would take a coil of very many turns to have an inductance of one henry, for use of smaller values it is customary to use such prefixes as micro or milli; for example, micro placed before a number changes its value to one one-millionth and milli changes it to one one-thousandth of its former value.

1 micro-henry $= \dfrac{1}{1,000,000}$ henry or in decimals .000001 henry.

1 milli-henry $= \dfrac{1}{1,000}$ henry or .001 henry.

The henry was once called the secohm, because a circuit having an inductance of one henry would only permit a rate of change of one ampere per second when the impressed current had an e. m. f. of one volt, and as the counter e. m. f. acted as one ohm resistance, we see from this that the inductance can be expressed in ohms.

The henry has also been called the quad or quadrant, because in the metric system a quadrant of the earth from the equator to a pole equals approximately 10^9 centimeters.   Both terms mentioned above are now quite obsolete.

As the inductance depends upon the flux interlinked with the current producing it, it is evident that by increasing the permeability of the medium surrounding the conductor, the flux is greatly increased and the inductance as well.

CHAPTER VI.

Although the manifestation of electricity is considered as moving at 186,000 miles per second, it does not necessarily imply that the current reaches its full value at the same speed. While it is practically true of non-inductive circuits, it is not true when a circuit contains **self-induction,** for when a direct current is applied to such a circuit it requires a certain time to bring the current up to its full value. It may take only an infinitesimal part of a second, or several minutes, as the case may be.

In the statement above it is not meant that there are true non-inductive circuits, as they do not exist, but it means that for ordinary purposes the self-induction is negligible. It must be understood that it is impossible to eliminate all inductance from a circuit.

When a direct current is applied to a circuit of high self-induction, it requires some time for the current to reach its maximum. When this point is reached, the circuit offers no more resistance to the flow of current than the true resistance of the conductor. If the circuit should be broken, however, the self-induction would again assert itself; this time not to oppose the flow of current, but to **prolong** it.

Because of this, the current is drawn out in a long flash at the break; which fact makes it difficult to break when large currents are carried. Of course, heavy currents produce flashes in circuits when broken, even though their self-induction is exceedingly small, as when a storage battery is short-circuited. But, because of the low voltage of the cell, the

spark or flash is very short, although it may be very hot. The resistance of the air suddenly becomes so high that the current stops.

But when the same cell is connected to a circuit of high self-induction and quite low resistance, the flash at the break becomes longer than before.

In explanation of this phenomenon we must take into consideration the magnetic field surrounding the electric circuit. We have already found that the current is opposed by the counter electromotive force generated by the expanding lines of force, and prolonged by the lines in their collapse upon the conductor. Thus the counter e. m. f. and impressed e. m. f. are in series when the circuit is broken. When a circuit of one turn has within it a flux $\Phi = 100,000,000$ lines when carrying one ampere of current, it would require one second for one volt to raise it to two amperes; or, to raise it from zero to one ampere, as the self-induction of such a circuit is one henry. Likewise, if the current were decreased or broken in one second of time, the resulting counter e. m. f. would be one volt, and as the impressed voltage is also one volt, it follows that two volts would have to be broken. The reason for this is that the 10^8 lines surrounding the single turn cut through it in one second as they collapse. If, however, the break were made in one-half a second, the counter e. m. f. would be two volts, as the 10^8 lines cut through it in one-half the time; or, at the rate of 200,000,000 lines per second. If the break takes place in $\frac{1}{1000}$ of a second, the counter e. m. f. would be 1,000 volts. The effect of inductance in an electrical circuit may be made clearer by means of a mechanical analogy.

When a very large fly-wheel is started from rest it requires a considerable length of time for the engine to bring it up to its normal speed, which fact is due to the fly-wheel's property of inertia.

Inertia, by definition, is the property of matter by which it tends to **preserve** a state of rest when still, and a state of motion when once started.

Energy is stored up in the fly-wheel because of this inertia, so that it continues to run some time after the steam is shut off. If there were no **friction** it would continue to run forever, but the energy is gradually dissipated in friction.

If a brake were used to **stop** the wheel, it would be exerting a pressure against the brake, which pressure would be very great if pressed very hard against the wheel; and, consequently, the energy stored up within it would soon be given off in **friction** at a high pressure.

In case the wheel were stopped **instantaneously**, it would be broken in pieces, as the pressure would become infinitely great to give up its energy so suddenly.

In comparison to self-induction we have what might be termed **electromagnetic inertia** of the ether.

Energy is stored up in the form of lines of force, so that the expansion of the lines can be considered parallel to the starting of the fly-wheel, and the magnetomotive force to the steam in the cylinder. The gradual breaking of the electrical circuit can be compared to the gradual stopping of the fly-wheel, as the electromagnetic energy is given off gradually at a low pressure; while the **sudden** breaking of the circuit would give off the energy very quickly at a high-pressure counter e. m. f.

In case the electrical circuit were broken instantaneously the counter e. m. f. would be infinitely high, as lines of force would be cutting through the conductor at an infinite speed.

The discovery of self-induction is given to the credit of Professor Henry, who published an account of his experiments under the heading, " Electrical Self-

induction in a Long, Helical Wire " (American Journal of Sciences, 1832).   Michael Faraday discovered the same phenomena in 1834, but laid no claim to precedence.   Professor Henry made his discovery of this extra current, as it was called, with his quantity magnet, heretofore mentioned.

When the natural resistance of an inductive circuit is negligible the inductive reactance in ohms is in direct proportion to the self-induction in henries divided by the time in seconds that has elapsed since a direct electromotive force has been applied to it.

Substituting X_s for inductive reactance in ohms, and L for inductance in henries and t for time in seconds, we have

$$X_s = \frac{L}{t} \text{ or } \frac{\text{henries}}{\text{seconds}} = \text{ohms.}$$

The counter e. m. f. manifests itself as **resistance,** which can be expressed in ohms as shown in the above equation; but, as there is no counter e. m. f. when the value of the current is **constant,** it is **evident** that there is also no reactance.   The value of the current is then governed only by the natural resistance of the conductor, and as it is assumed in the above that there is practically no resistance, it is plain that there is almost no limit to the value the current would reach if allowed time sufficient.   As the reactance is expressed in ohms, it follows that we can apply Ohm's law by substituting

X_s for R in the equation $I\frac{E}{R}$ so that it becomes $I\frac{E}{X_s}$ or $I = \frac{E}{\left(\frac{L}{t}\right)}$

In a circuit of practically no natural resistance, having an inductance of one henry, one volt of direct E would cause a current to increase from zero to one

ampere at the end of one second, as shown in the equation $\frac{\text{one henry}}{\text{one second}} = 1$ ohm reactance, and $\frac{\text{one volt}}{\text{one ohm}}$ = one ampere.

The foregoing is shown graphically in Fig. 48, where the vertical lines, or ordinates, represent the value of the current and the horizontal lines, or abscissas, the time the direct e. m. f. is applied. By **increasing** the inductance L the rise of current is retarded more than in the first case; so, when we substitute **two** henries for one as in the preceding, we find that it takes one volt two seconds to bring the value of the current up to one ampere, and four seconds to two amperes, and so on.

The **energy** required to raise the current to certain values in purely inductive circuits is not lost but is stored up in the form of magnetic flux, interlinked with it. This energy is restored when the impressed e. m. f. is removed or decreased. In which case it manifests itself in counter e. m. f.

As the counter e. m. f. depends upon the rate of change of the flux surrounding the circuit, it is clear that when the circuit is broken suddenly, the counter e. m. f. is very high and the energy is given off very quickly.

On the other hand, when the current in the circuit is **diminished** slowly, the counter e. m. f. is not as high as before, but continues for a longer period of time, the energy given off in both cases being the same.

Since the counter e. m. f. is one volt when the current is varied at the rate of one ampere per second when the inductance L is one henry, it can be seen that the counter e. m. f. is directly proportional to the inductance L, or $L = E_c$, and the power at the end of one second can be expressed by $L\ I =$ watts when the current varies one ampere per second.
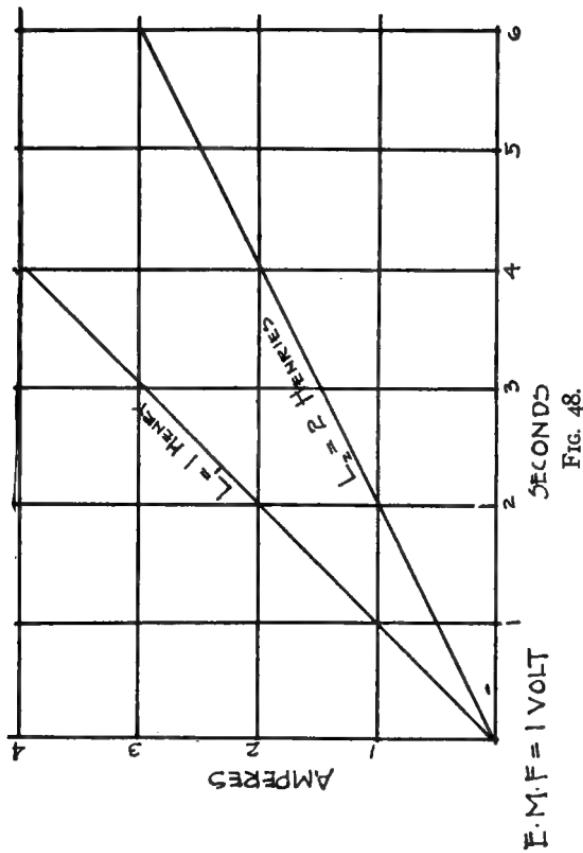
FIG. 48.

But when the rate of current variation is **increased,** the counter e. m. f. accordingly increases so that it equals $L \dfrac{I}{t} = E_c$, As the work in joules or watt-seconds depends upon the value of the current and the time t the counter e. m. f. is engaged, so

$L \dfrac{I}{t} I t = L I^2 =$ joules. The current, however, is not constant but is assumed to rise uniformly from zero to its certain value, as shown in Fig. 48, therefore $\dfrac{L I}{2} =$ joules or watt-seconds.

If it is desired to create a current of ten amperes in a circuit of one henry, we find that the work required is $\dfrac{1 \times 10^2}{2} = 50$ joules or watt-seconds.

This answer can be converted into watt-hours by dividing by 3600.

CHAPTER VII.

As it may be desired to know the time required for a direct e. m. f. to bring a current up from zero to a desired value in a purely inductive circuit, it can be obtained by the equation $\dfrac{L}{\frac{E}{I}} =$ time in seconds where I is the desired current.

The equation used to determine the growth of current in a circuit containing resistance is derived by the use of higher mathematics, which will not be given, as it is not in the province of this treatise to do so. The equation itself, however, can be used to advantage as it is purely arithmetical.

The celebrated Von Helmholtz deduced the following formula: $i = \dfrac{E}{R}\left(1 - \epsilon^{\frac{-Rt}{L}}\right)$

This formula presents a formidable appearance but its solution is not as difficult as it appears to be.

The first part of the formula we recognize as Ohm's law, giving us the final value of the current when a direct e. m. f. is applied to an inductive circuit.

As the extra current exerts opposition to the impressed current, it is clear that at any instant the current has reached a value which is the difference between the final value and the value of the current opposing it. As the counter e. m. f. E_o forces a cur-

rent through the same resistance as the impressed e. m. f. E, it follows that their difference is $\frac{E}{R} - \frac{E_o}{R} = i$ or $\frac{E-E_o}{R} = i$.

The difference between the impressed e. m. f. and the counter e. m. f. represents the effective e. m. f. acting upon a circuit as though containing only resistance.

When the impressed current is increasing from zero to its final value, the extra current is a fraction of this final value as shown by the term $\epsilon^{\frac{-Rt}{L}}$ The Greek letter **epsilon** ϵ is a symbol of 2.71828; or, approximately 2.718.

The exponent in this term is fractional and also preceded by a minus sign. Their significance will be made plain in a subsequent solution of a problem.

This fraction times the final value of the current equals the extra current i_o produced by E_o, thus $i_o =$

$$\frac{E}{R} \times \epsilon^{\frac{-Rt}{L}}$$

The real current at any instant equals $I - i_o = i$, where I equals the final value. A close inspection of the complete equation will reveal the preceding operations for $i = \frac{E}{R}\left(1 - \epsilon^{\frac{-Rt}{L}}\right)$ is the same as $i = \frac{E}{R} -$

$\left(\frac{E}{R} \times \epsilon^{\frac{-Rt}{L}}\right)$ where the first term represents the final value of the current and the second term, the extra current.

In a circuit having an inductance L of 3 henries, e. m. f. E of 8 volts and a resistance R of 2 ohms, we can find the growth of current after two seconds by substituting these values in the equation,

$$i = \frac{8}{2} \left(1 - 2.718^{\frac{-2 \times 2}{3}} \right)$$

The term $\frac{-2 \times 2}{3}$ or $-\frac{4}{3}$ is the exponent of 2.718. For the benefit of those who are not familiar with fractional exponents it will be remarked that the upper number or numerator (4 in this case) indicates that 2.718 must be raised to its fourth power, that is, multiplied by itself four times. The denominator 3 indicates that the cube root should be extracted from the fourth power of 2.718, or $\sqrt[3]{2.718^4}$

The minus sign preceding the exponent indicates that it is the **reciprocal** of a positive exponent, in other words the root of the power of the number divided by one, as in the following

$$2.718^{\frac{-4}{3}} = \frac{1}{\sqrt[3]{2.718^4}} = \frac{1}{\sqrt[3]{54.575}} = \frac{1}{3.79} = .26385$$

The formula then appears

$$i = \frac{8}{2} (1 - .26385) = 4 \times .73615 = 2.9446 \text{ amperes}$$

∴ the current has reached a value of 2.9446 amperes in 2 seconds after contact. It is evident that when the inductance L (the denominator of the fractional exponent) is large, it becomes very difficult to extract the root arithmetically, and the handling of very large numbers is at once both slow and inconvenient.

A much quicker method to solve problems of this nature is offered in the use of **logarithms** which may be learned in a very short time.

The ease with which such problems can be solved is seen in the solution of the same problem by logarithms.

To obtain $2.718^{\frac{-4}{3}}$ we first find (in a table of logarithms) the logarithm of 2.718, which is .434249. This is next multiplied by the fraction $\frac{4}{3}$ which gives us the logarithm of the cube root of the fourth power of 2.718 or $\frac{4}{3} \times .434249 = .57898$.

Again referring to the table we find that .57898 is close to the logarithm of 3.79 $\therefore$ 3.79 is the $2.718^{\frac{4}{3}}$

Because of the minus sign preceding the exponent, 3.79 is made the denominator and one the numerator, so $\frac{8}{2}\left(1-\frac{1}{3.79}\right) = 4\left(1-\frac{1}{3.79}\right) = 4-\frac{4}{3.79} = 4-1.056 = 2.944$ amperes.

By working Helmholtz's equation a great number of times and changing the value of t by small intervals each time, from zero until the final value of the current is reached, a curve can be plotted showing the growth of the current graphically. See Fig. 49.

The e. m. f. is 10 volts in both circuits shown in the diagram.

We come to the conclusion while inspecting the curves, that the natural resistance of the circuit determines the maximum value of the current but does not oppose the growth of current, while the inductance does not determine the final value but opposes the growth.

FIG. 49.*
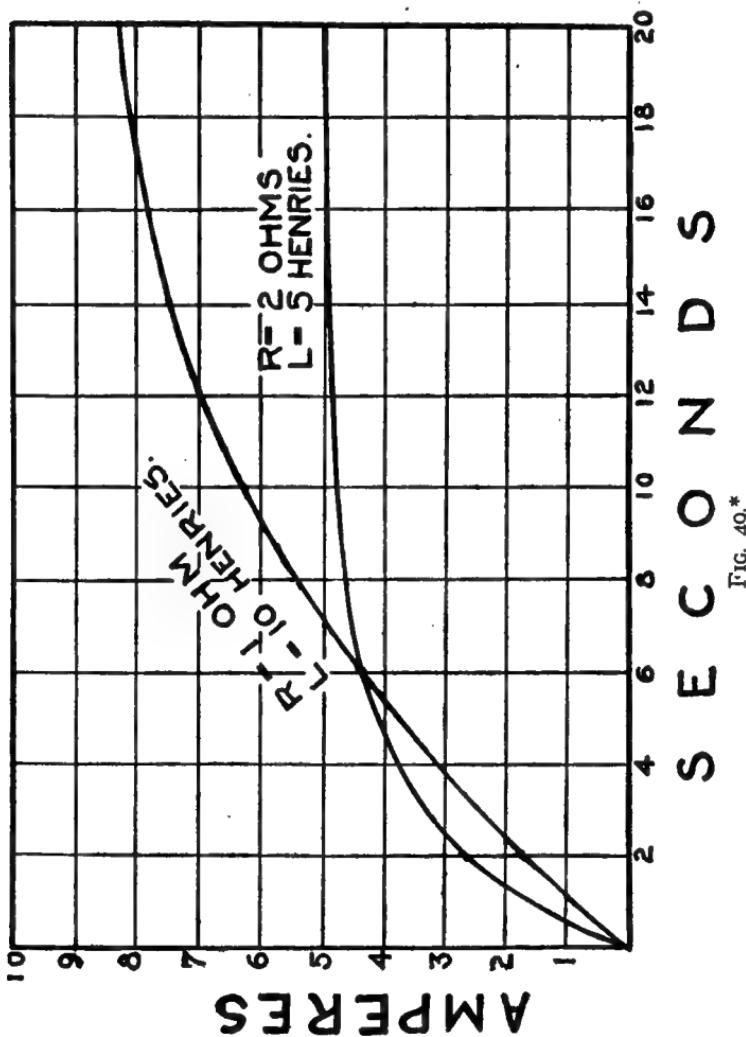
In alternating-current circuits the e. m. f. is not constant, but is ever-changing in value so that the preceding would not hold good.

In considering the first half of a wave of e. m. f. from zero to maximum, we see that in summing up the effect of the varying e. m. f., the current would increase more slowly than when the e. m. f. is constant at the maximum value. At this point the current may have reached only a fraction of its maximum value, but continues to increase, but still more slowly as the e. m. f. decreases, so that the current reaches its maximum some time after that of the e. m. f. This is the wave during which the energy is first stored up in the circuit which can be termed the initial current of the inductance.

When the e. m. f. is zero the current begins to decrease until it reaches zero some time after the e. m. f.

This phenomenon is termed the **lagging** of the current behind the e. m. f. producing it. As the current in an inductionless circuit increases and decreases at the same time as the e. m. f. which produces it, such current is said to be **in phase** with the e. m. f.

Thus one portion of the e. m. f. E_R can be considered as being consumed in resistance and another portion E_X (Fig. 50) consumed in sending current against the counter e. m. f. of inductive reactance. As the counter e. m. f. depends upon the rate of change of the flux surrounding the circuit, and as this is greatest when the current is passing through zero, the counter e. m. f. is also greatest then.

As the number of degrees the e. m. f. is ahead of the current consumed in inductance is 90°, they are said to be in **time quadrature.**

Thus, the latter portion of the e. m. f., E_X, must be maximum when the e. m. f. consumed in resistance is

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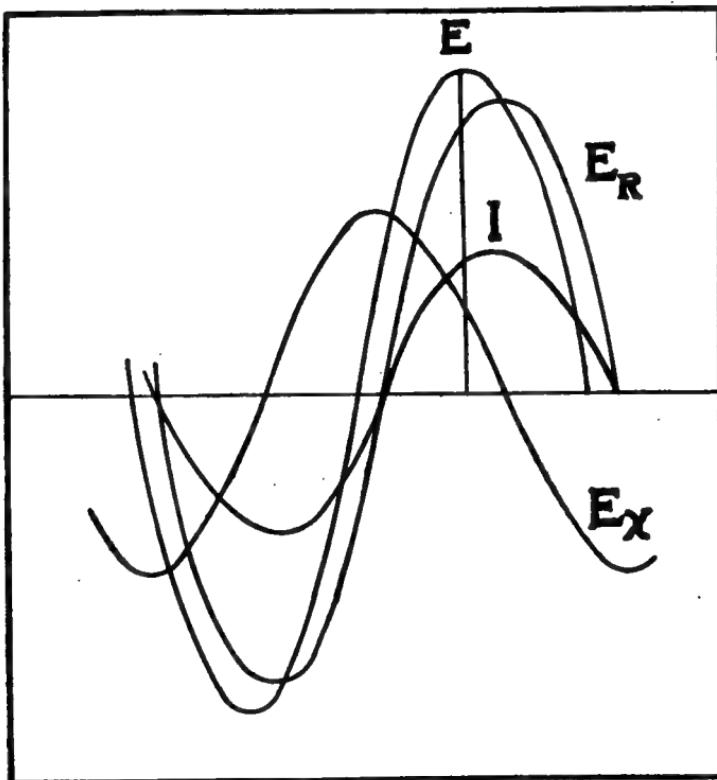
FIG. 50.

zero. The required e. m. f. to force a certain current through such a circuit is shown as E, the impressed e. m. f. which is the sum of the e. m. f.'s E_R and E_x at the same instant. Referring to Fig. 50 it is seen that this resultant e. m. f. E is ahead of the current by a number of degrees, which is termed the **angle of lag** of the current.

CHAPTER VIII.

The power consumed in an alternating circuit is not obtained **directly** by the product of the e. m. f. and the current, as in direct-current circuits, because the e. m. f. and current are continually **changing** in value.

In case the circuit contains inductance, another difference exists which is due to the lagging of the current behind the e. m. f. producing it.

The product of the volts and amperes for every instant during one cycle forms a new curve, which represents the changing value of power in watts in the circuit during one cycle. One form of such a curve of watts in a circuit containing only **resistance** (as an incandescent lamp circuit), is shown in Fig. 51.
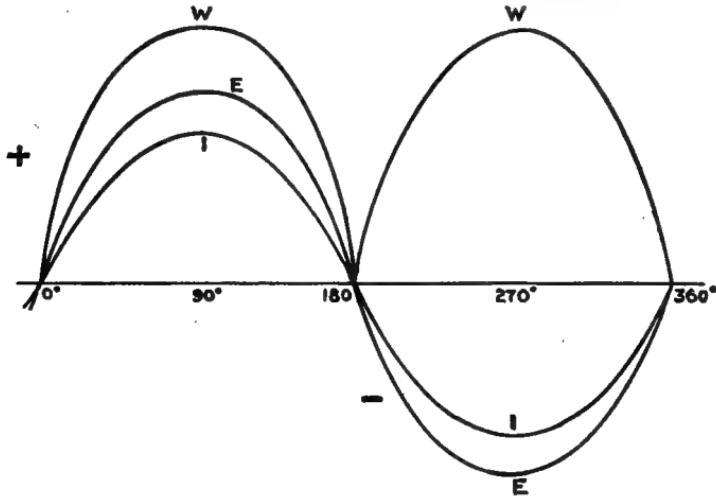
FIG. 51.

As the volt E and amperes I are both zero at 0°, their product at that instant would also be zero watts W, and as the volts and amperes are both maximum at 90°, the watts would also be maximum; then, while at 180° the volts, amperes and watts would be zero again.

At this point the e. m. f. is changed in polarity and remains so until the end of the cycle, consequently the current which it produces is reversed and flows in this direction until the end of the cycle. It is clear that as much power is consumed in forcing the current through the resistance in one direction as another, so we see that as much power is consumed during the last half of the cycle as during the first half.

The power wave when placed above the line indicates that power is being consumed, but when placed below the line it indicates that energy which has been stored up in the circuit is being returned to the generator. As all the power consumed in resistance is dissipated as heat, as in Fig. 51, no power is returned, thus both halves of the power wave in Fig. 51 are placed above the line.

By examination of Fig. 52 it will be seen that as there is a phase difference of 90° between the e. m. f. and current, there are six points of zero value during one cycle. The first occurs when the e. m. f. begins, the second when the current begins, the third when the e. m. f. completes half a cycle, the fourth when the current completes half a cycle, the fifth when the e. m. f. ends its cycle, and the sixth zero occurs when the current ends its cycle. As the product of the e. m. f. and current at the instant either one of them is zero would be zero no matter how great the value of the other factor, it is clear that the power wave must pass through zero six times The power waves above the line W + W +, and those below the line W — W —, are equal, thus the power consumed by
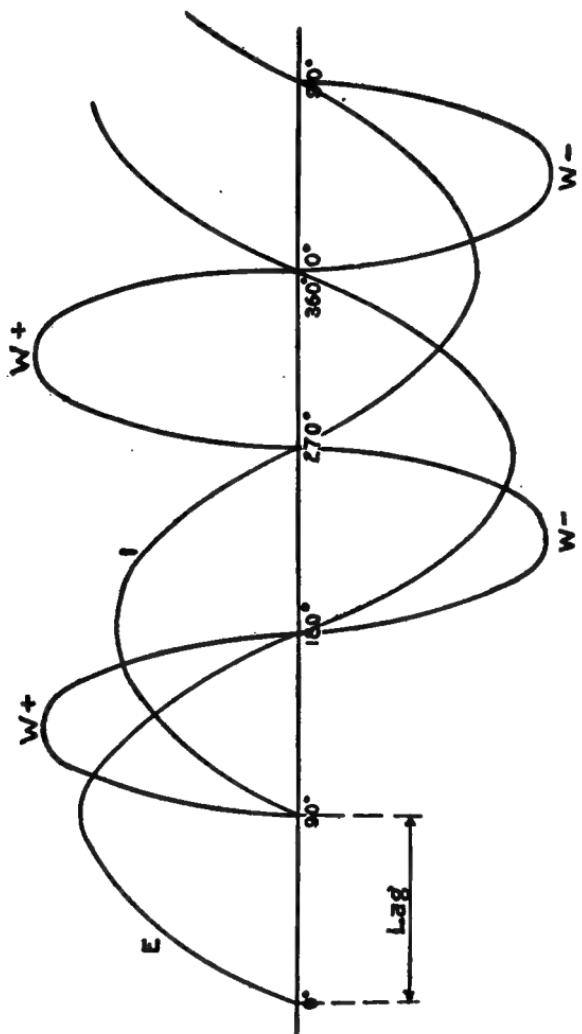
FIG. 52.

the circuit during the time the e. m. f. is progressing from 90° to 180° is returned to the generator during the time the e. m. f. is progressing from 180° to 270°, and the power consumed during the time the e. m. f. is progressing from 270° to 360° is returned to the generator during the time the e. m. f. is progressing from 0° to 90° in a new cycle. These changes take place in a purely **inductive** circuit. As the current pulsates through such a circuit without **being** used up or dissipated, it is termed a **wattless current.**

Power is consumed in both resistance and inductive reactance.

In Fig. 50, E_R is the portion of the e. m. f. consumed in resistance, and I_R the portion of the **current** consumed in resistance. The product of their instantaneous values produces a curve which represents the portion of power consumed in resistance.

In the same way the product of E_X, the portion of the e. m. f. consumed in reactance, and I_X, the portion of the current consumed in building up a magnetic field around the circuit, produces a power curve which represents the power consumed in reactance.

E_R and I_R are termed the **power or watt components,** and E_X and I_X are termed the **idle or wattless components.**

The power wave W+ in Fig. 53 represents the power consumed in a circuit containing both resistance and reactance, while the wave W— represents that the portion of the power consumed in reactance is returned to the generator; therefore, the power really used up is the **difference** between W+ and W—.

The greater the difference in phase between the e. m. f. and current the greater is the wattless current, therefore less power is used up.

Their difference in phase can never be greater than 90°. When the current lags 90° behind the e. m. f. the **power factor** is said to be **zero,** as all the
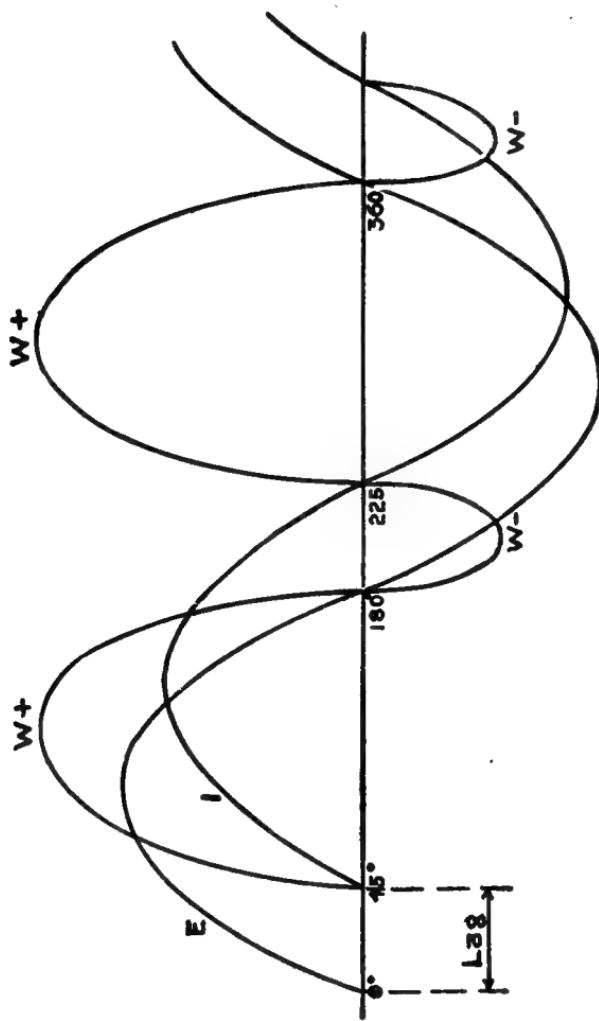
FIG. 53.

power consumed by the circuit is returned to the generator. When the e. m. f. and current are in phase the power factor is **one,** as none of the power consumed is returned.

The reactions of the magnetic field will now be described in greater detail in connection with Fig. 54. It would be untrue to consider that the current in a purely inductive circuit I_X remains at zero until the e. m. f. progresses 90° when we consider the very first half-cycle; for, in case it is during the first half-cycle, the current I_X will start at the same instant as the e. m. f. E_X, but it does not increase as rapidly as E_X but continues in growth until E_X reaches zero at 180°, at which point the current I_X is maximum.

The first half-cycle of I_X is somewhat higher at maximum than it is at maximum at any succeeding cycle, because it is under the influence of E_X longer during the first half-cycle. After the first half-cycle the current I_X lags 90° behind the e. m. f. E_X.

If we consider the helix in Fig. 55 as the inductive circuit under consideration in Fig. 54, we observe that the flux threads through it in a direction depending upon the direction of the current at any instant. We will assume the current as entering the helix at the turn marked 5 and leaving it at the turn marked 1 during the progress of the e. m. f. E_X from 90° to 180°. The flux then created in the coil or helix has a direction shown by the arrows in the serrated lines. The **density** of this flux Φ increases directly with the current, in other words Φ is in phase with the current I_X, Fig. 54.

As the flux Φ increases it **expands.** During this expansion it induces a counter e. m. f. E_C which opposes E_X and consequently the growth of the current I_X.

When E_X reaches zero, I_X and Φ are maximum and the expansion stops, thus E_C becomes zero at that

See Fig 54   See Fig 55   See Fig 56   See Fig 57
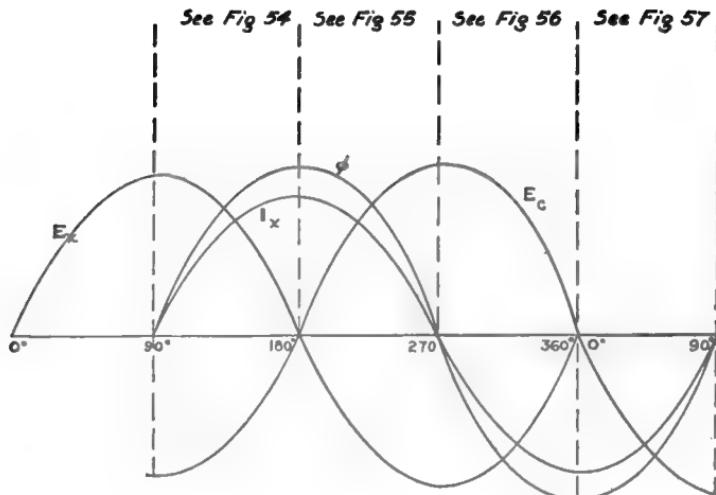
FIG. 54.

instant. All the energy in the circuit is now magnetic.

As E_X at that instant exerts no effort upon I_X and therefore upon Φ, the flux collapses (Fig. 56). The direction of the flux remains the same, but the movement across the turns is reversed; thus the E_C is generated with a reversed polarity, so that instead of opposing the growth it tends to keep the current I_C flowing until all the energy stored up in the Φ is returned to the generator. E_X has been increasing in the meantime and opposing the action of E_C until Φ and I_X reach zero, at which instant E_X reaches maximum. E_C also reaches maximum at this instant because the greatest rate of change in flux density takes place while Φ passes through zero.

At this instant E_X introduces a current through the helix in the reverse direction from that shown in

Figs. 55 and 56. The flux which this current creates is also reversed as shown in Fig. 57.

The flux is here expanding as its density increases, consequently E_C is generated. The polarity of E_C is the same during the expansion of this flux as the collapse of the flux in Fig. 56, therefore E_C would exert opposition to E_X. E_X and I_X, however, store up energy in Φ from the time E_X has progressed from 270° to 360°, which energy is returned from 360° to

Current I_x up

Current I_x down

Direction of expansion
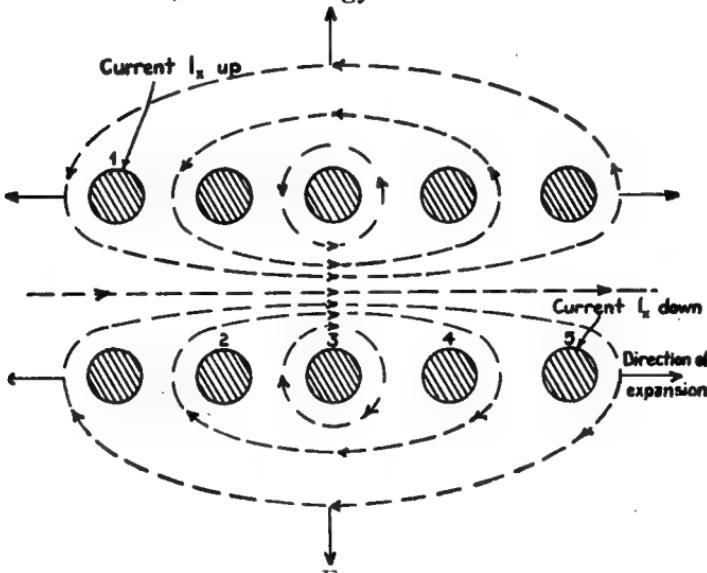
Fig. 55.
FLUX EXPANDING.

90° of the new cycle. The flux at 360° collapses, as at this instant E_X is zero again.

In its collapse (Fig. 58) it generates E_C of the reverse polarity of E_C in Fig. 57, because the direction of the flux remains the same, but its movement across the turns during collapse is then opposite.

CHAPTER IX.

The resistance produced by the counter e. m. f. in an inductive circuit is generally of more importance than the lagging of the current. The maximum value of the flux threading through the turns of a helix represents the total number of lines created in it by the current as it increases from zero to maximum. This total number of lines move across and consequently are cut by the turns four times during every cycle, as will be noticed by inspection of Figs. 55, 56, 57, 58.

Current I_x up

Current I_x down

Direction of Collapse
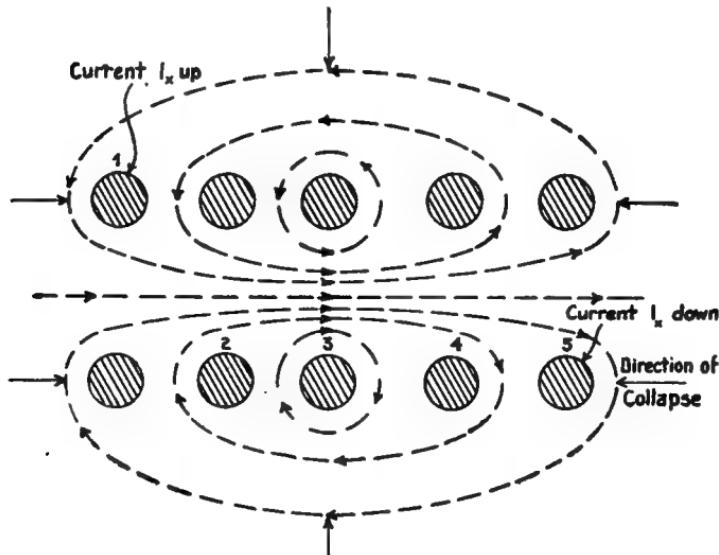
FIG. 56.
FLUX COLLAPSING.

If it takes one second for one cycle of flux changes in the foregoing, the average number of lines cut by the turns S per second is 4 Φmax. S.

Take for example a helix of five turns so close together that practically all the flux interlinks with all the turns.

Current I_n down

Flux reversed
Current I_n up
Direction of expansion
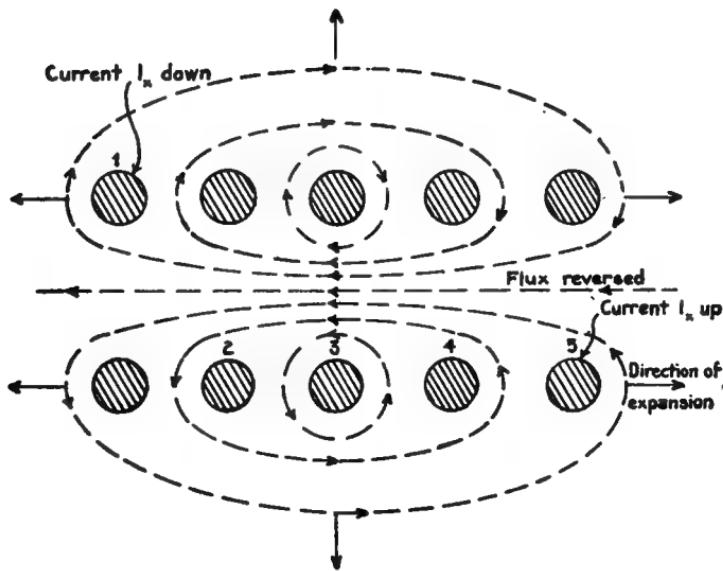
FIG. 57.

At the same frequency of one cycle per second, with a maximum flux of 1,000 lines, the average number cut per second is 4 × 1,000 × 5 = 20,000 lines per second.

As it requires a rate of cutting of 10^8 lines per second to generate 1 volt, the preceding product can be converted to volts by dividing 20,000 by 10^8 = .0002 volt.

The formula would then be written:

$$\text{Average } E_c = \frac{4 \, \Phi\text{max. S}}{10^8}$$

If the frequency is increased to sixty cycles per second, the changes of flux take place sixty times as fast as before, so we find that the

$$\text{Average } E_c = \frac{4 \times 1,000 \times 60 \times 5}{10^8} = .012 \text{ volt.}$$

Then we write the formula:

$$\text{Average } E_c = \frac{4 \, \Phi\text{max. f. S}}{10^8}$$

To make it plain that the average generated e. m. f.

Current I_x down

Flux reversed
Current I_x up

Direction of
Collapse
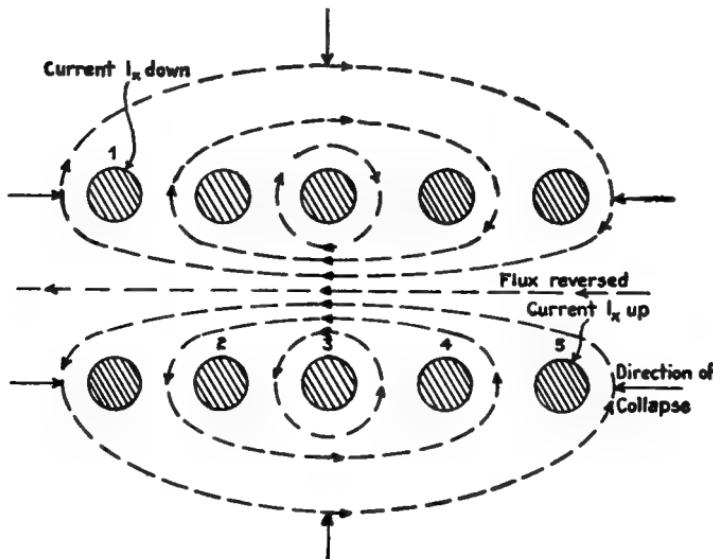
FIG. 58.
FLUX COLLAPSING.

is the same whatever the form of wave, as long as the maximum flux is the same we will consider that a conductor is moved across a uniform magnetic field at different speeds.

As the result is the same whether the conductor is moved across the flux or the flux is moved across the conductor, we see that the different rates of speed of the conductor really represents the different rates of speed at which the flux moves across a conductor in its expansion and collapse.

In Fig. 59 the flux shown by a number of parallel lines is considered as the maximum number of lines in a helix, spread out into a uniform magnetic field of 10^8 lines of force.
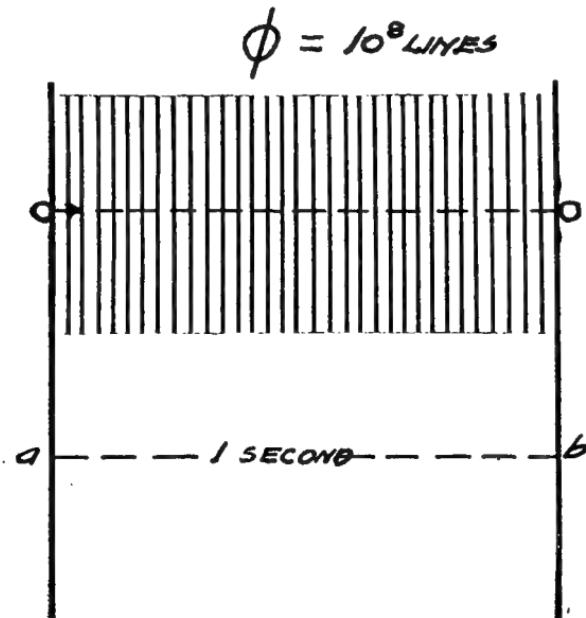
$$\phi = 10^8 \text{LINES}$$

FIG. 59.

If the conductor moves at a uniform rate of speed from **a** to **b** in one second, it cuts lines at every instant at the rate of 10^8 lines per second, which generates an e. m. f. of 1 volt. In this case the maximum and average voltage are the same, as is shown in Fig. 60.
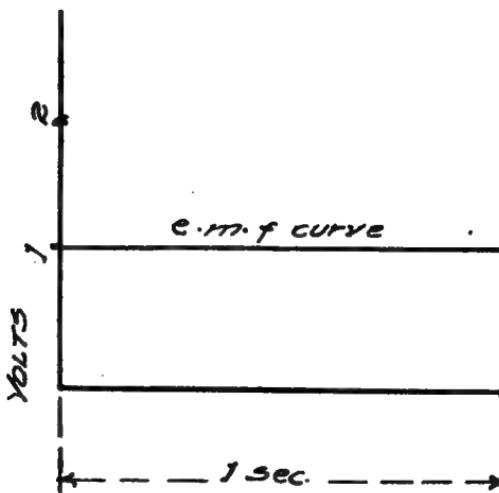
FIG. 60.

If the conductor, Fig. 61, moves from **a** to **b** in ⅕ second, it cuts ⅕ of 10^8 lines, or at the rate of $5 \times$ ⅕ of 10^8 lines per second, or 33,333,333 lines, which generates an e. m. f. of ⅓ volt.

In moving from **b** to **c** it cuts twice as many lines in ⅕ a second, thus the e. m. f. generated is doubled, or ⅔ volt.

From **c** to **d** the movement is three times as fast as from **a** to **b**, so the e. m. f. is 1 volt; from **d** to **e**, four times as fast as from **a** to **b**, so the e. m. f. is 1⅓ volts, and from **e** to **f**, five times as fast as from **a** to **b**, so the e. m. f. is 1⅔ volts.
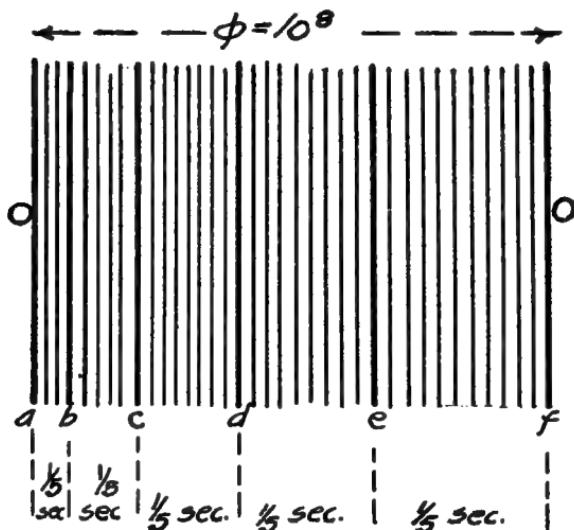
Fig. 61.

• The wave form of these changes of e. m. f. is shown in Fig. 62.

The average value of e. m. f. of such a wave is obtained by adding all the values during equal divisions of time and dividing their sum by the number of divisions taken.

As the divisions of time are each ⅕ second, and the consecutive values of e. m. f. are ⅕, ⅖, 1⅕ and 1⅖ volts, respectively, their sum, or 5 volts, divided by the five divisions of time, gives 1 volt as the average. The maximum voltage is 1⅖ times the average in this example.

If the conductor, Fig. 63, is moved uniformly from **a** to **b** in one second in a path shown by the curved line, it is clear that it must move faster than in Fig. 59.
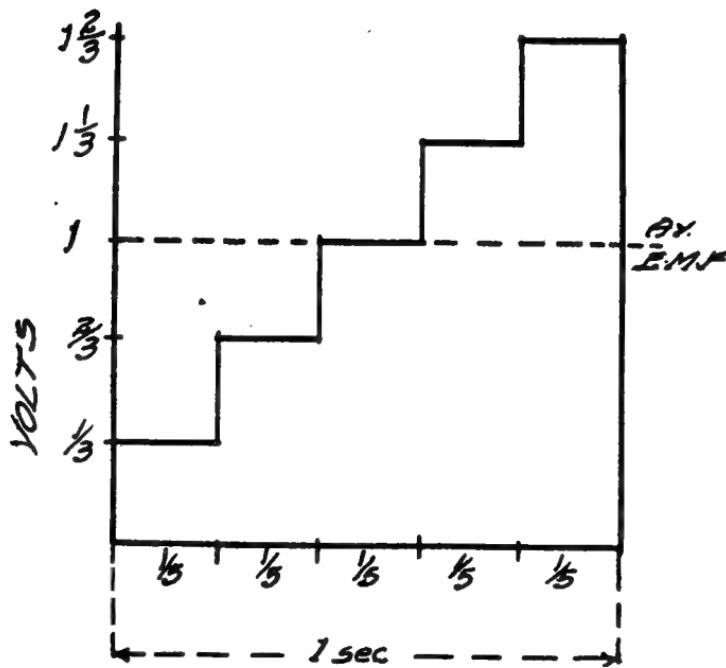
FIG. 62.

When the movement is nearly at right angles to the flux, the e. m. f. is, of course, higher than in Fig. 60.

The wave form of the generated e. m. f. in the conductor is seen to be the first quarter of a sine wave, Fig. 64. To obtain the average e. m. f. of such a wave involves higher mathematics, but the principle is to divide the wave into small steps of value from zero to maximum.

The time line is divided into an indefinite number of equal divisions which correspond to the small steps of e. m. f. This is shown roughly in Fig. 64.

9

The sum of all the successive values of e. m. f., from zero to maximum, divided by the number of divisions of time, gives the **average** value. In our consideration of Figs. 60 and 62 we found that the average e. m. f. is the same whatever the wave form, so like-

$$\phi = 10^8$$
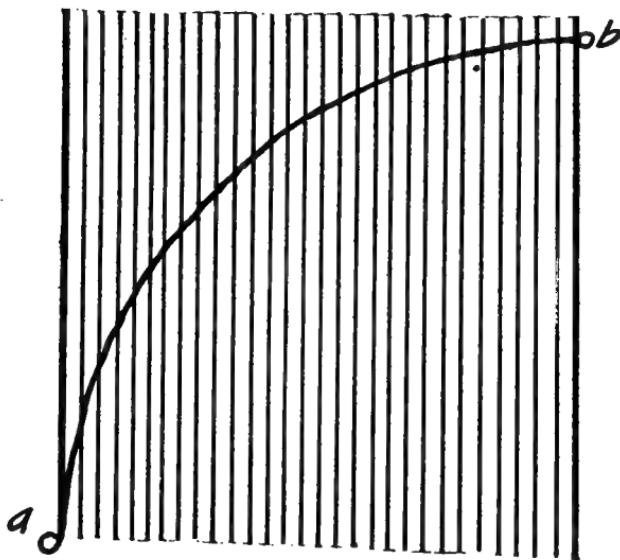
FIG. 63.

wise the average value of the e. m. f. in Fig. 64 is 1 volt.

It was stated before that the conductor in Fig. 63, moving at a uniform velocity from **a** to **b** in one second, must move faster than when moving straight across the flux in one second. As the curved line, Fig. 63, represents 'one-quarter of a circular path, a path

straight across the flux, as in Fig. 59, would compare in length to the radius of the circular path. Thus, the velocity of the conductor moving in the curved path compares with the velocity of the conductor moving in the straight path as the length of one-quarter of the circumference compares to the radius.

The circumference of the circular path is 2π times the radius, thus one-quarter of the circumference is equal to $\frac{2\pi}{4}$ or $\frac{\pi}{2}$ times the radius; and, conversely the radius is $\frac{2}{\pi}$ times the length of one-quarter of the circumference.

Then we deduce that the velocity of the conductor in Fig. 63 is $\frac{\pi}{2}$ times the velocity of the one in Fig. 59; or, to put it the other way, the uniform velocity straight across the field in Fig. 59 is $\frac{2}{\pi}$ times the velocity of the conductor in Fig. 63.
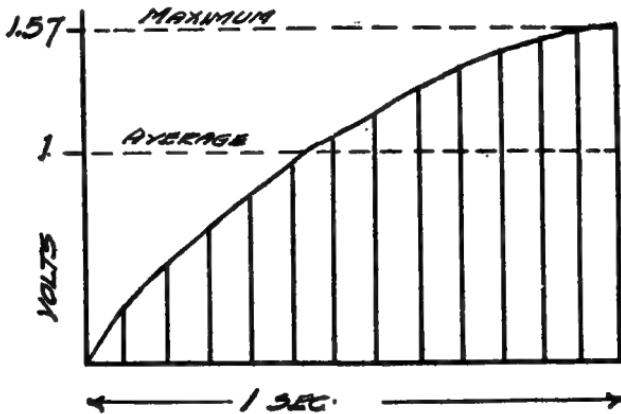
Fig. 64.

When the conductor in Fig. 63 leaves **a** in its movement to **b**, it at first moves more along the length of the lines than across them, but when it reaches **b**, it moves at right angles to them.

At this point the most effective, and as the velocity across the flux is here $\frac{\pi}{2}$ times that in Fig. 59, the e. m. f. generated is $\frac{\pi}{2}$ times that generated in the conductor in Fig. 59; or, $\frac{\pi}{2}$ times one volt which equals 1.5708 volts, the maximum value as shown in Fig. 64.

When the maximum voltage of a sine wave is known, the average value is $\frac{2}{\pi} \times$ E max.

We found that the average counter e. m. f. is $\frac{4 \Phi \text{ max. f. S}}{10^8}$ in volts, so the maximum value is $\frac{\pi}{2} \times$ $\frac{4 \Phi \text{ max. f. S}}{10^8} = \frac{2 \pi \Phi \text{ max. f. S}}{10^8}$.

In the formula for **self-induction** which is L $=$ $\frac{S \Phi}{I \, 10^8}$ in henries, the current factor I is amperes, and $I = \frac{S \Phi}{L \, 10^8}$.

It will be observed that the flux in the latter formulæ is not represented as being the maximum value, as it is not indicating a varying flux but the total lines enclosed by the helix, in a stationary condition.

As the maximum value of the flux is really the total number of lines created by the current from **o** to max-

imum, they can be considered identical in these equations.

We find in one equation the maximum value of the counter e. m. f. and another the current value, so by using Ohm's law we can obtain the reactance $X_s =$

$$\frac{2\,\pi\,\Phi\,\text{f. S}}{10^8} \div \frac{\text{S}\,\Phi}{\text{L}\ 10^8} = \frac{2\,\pi\,\Phi\,\text{f. S}}{10^8} \times \frac{\text{L}\ 10^8}{\text{S}\,\Phi} = 2\,\pi\,\text{f. L;}$$ react-

ance in ohms.

CHAPTER X.

The self-induction of a coil or helix can be greatly increased by introducing an iron core into it. The **higher** permeability of the iron, compared with air, permits a correspondingly greater flux to be enclosed by the coil, consequently a higher self-induction. In this way it is possible to have a coil of very low resistance, but of very high inductance. A direct e. m. f. of 1 volt may force as much current through such a coil as a 100 or more volts alternating current with a proper choice of frequency.

In Fig. 65 is shown a coil of wire in series with an

Lamp bright

A.C.

FIG. 65.
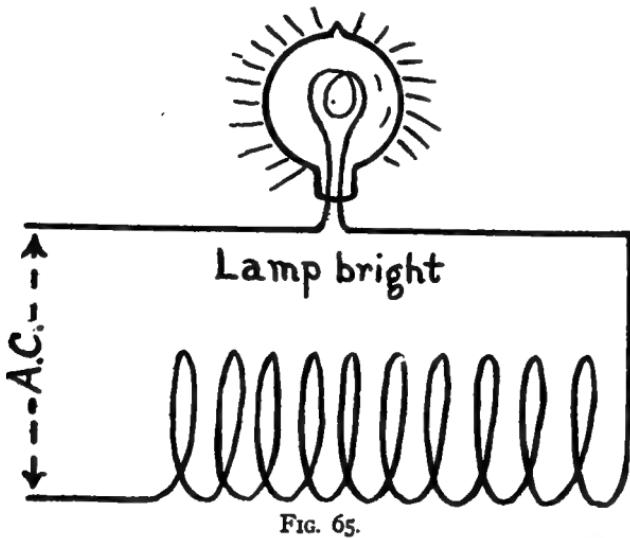
incandescent lamp. The lamp burns at almost full candle-power, as the inductance of the coil is very small.

In Fig. 66, however, the light gets dim, and grad-

Lamp dim.

A.C.
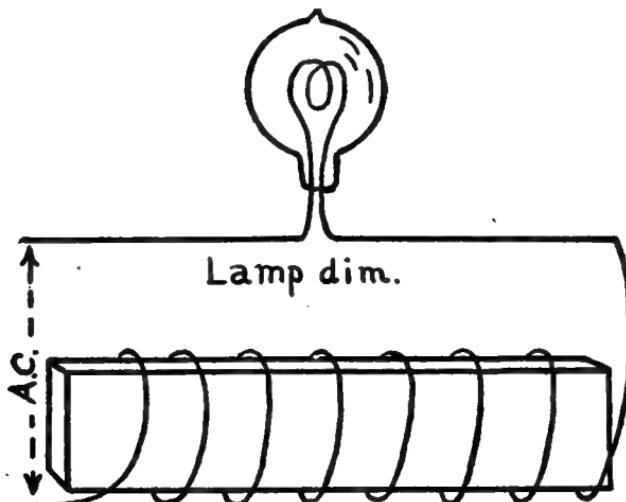
FIG. 66.

ually goes out as the iron core is inserted, as now the **inductance** is very high.

Not only can incandescent lamps be controlled by means of a variable-inductance coil, but also transformers and other kinds of electrical apparatus; thus, such a coil can be made to take the place of a **rheostat** with its waste of current.

Various methods are used to vary the inductance of such coils, such as: (1) **sliding contacts,** which vary the number of turns upon a stationary iron core; (2) a movable **coil,** and (3) a movable **core.**

The first method has the advantage of very high

Handle

To A.C.
Mains

to
apparatus
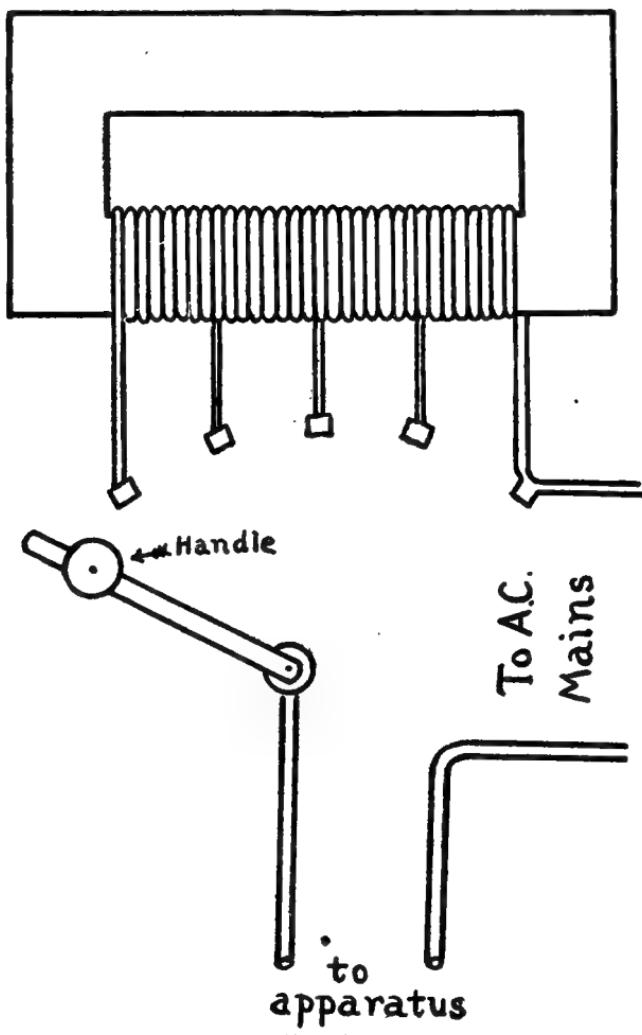
Fig. 67.

efficiency, as the magnetic circuit can be made as perfect as possible. It has the disadvantage of varying the current in steps and of sparking at the contacts. One form of such a coil is shown in Fig. 67.

The second method is not used very much, owing to the difficulty in handling or supporting the coils.

The coils in Figs. 66, 68, 69 come under the third method, which is most commonly used.
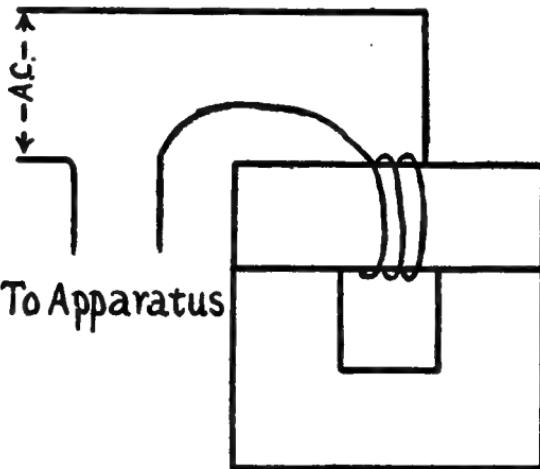
FIG. 68.

The coil in Fig. 66 is wound on a long, square paper or fiber tube. The core, which is made up from strips of sheet iron or steel, should move freely in the tube so that it can be drawn out or inserted uniformly; and, in so doing, vary the current uniformly. Such an inductance coil has the advantage of simplicity and uniform regulation, but has the disadvantage of low efficiency and of the tendency of the core to be pulled into the tube when large currents are

controlled.   Many more turns are required in such a
coil than in the two following, which have the advan-
tages of high efficiency and a very uniform and wide
range of regulation.

The cores in each of these methods are made in two
parts: a stationary, which supports the coil, and a
movable, called the yoke, which is introduced into the
coil in Fig. 68, and which lies across the ends of the
stationary part shown in Fig. 69.

To Apparatus.
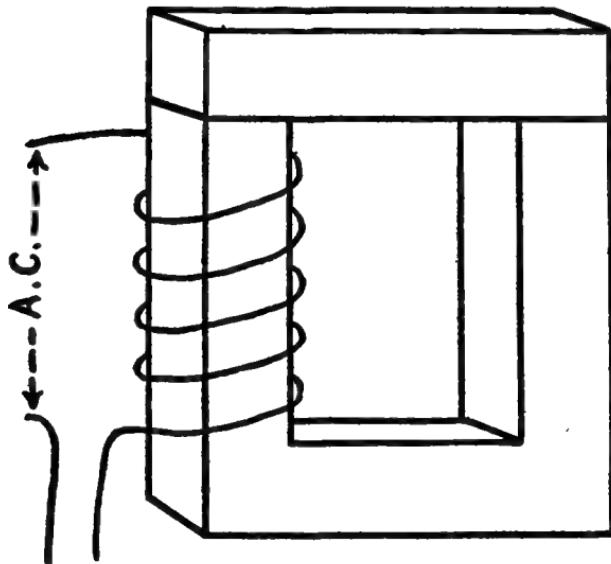
FIG. 69.

By moving the yoke on or off the end enclosed by
the coil in Fig. 69, nearly perfect regulation is secured.
It will be observed the coil has considerable self-

induction left, even when the yoke is removed, as the coil still surrounds one end of the core. To remedy this, the coil is tapped at two or three points; thus connection can be made at any one of the taps before operating. Connection should not be made or broken at the contacts when the current is being used.

Coils as described are generally termed reactance coils, and sometimes **compensators.** They will be found very useful and economical to those who have alternating current to control.

In graphically representing reactance and resistance, it is customary to indicate them by a spiral and zigzag line respectively. As the reactance in ohms, $X = 2 \pi f L$, it is possible to obtain the value of the inductance by the formula:

$$L \text{ in henries} = \frac{\text{reactance in ohms}}{2 \pi \text{ frequency}}, \text{ or } \frac{X}{2 \pi f}$$

In case a coil whose resistance is negligible and of unknown self-induction is connected to 110 volts a. c. at 60 cycles, and the coil permits only ½ ampere to flow, the reactance in ohms is $\frac{110}{\frac{1}{2}} = 220$ ohms.

Thus, $L = \dfrac{220}{2 \times 3.1416 \times 60} = .5835+$ henries (Fig. 70).

When considerable resistance is present, the com-

$$f = 60 \qquad L = .5835$$
$$X = 220 \text{ ohms}$$
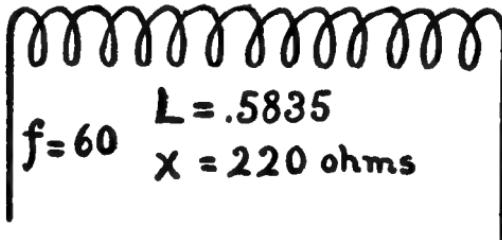
FIG. 70.

bined effect of it and the reactance is not exactly equal to the sum of their individual resistances in ohms, as the e. m. f. consumed in inductance is in time quadrature with the e. m. f. consumed in the natural resistance.

Their combined effect in ohms, called the **impedance,** whose symbol is Z equals $\sqrt{R^2 + (2\pi f L)^2}$, but as $2\pi f L = X$, it can be written $Z = \sqrt{R^2 + X^2}$.

Consider the coil in the previous problem as having a natural resistance of 3 ohms, then

$Z = \sqrt{3^2 + 220^2} = \sqrt{9 + 48400} = 220.02$ ohms.

It is evident from this answer that the **apparent** resistance is governed almost entirely by the inductance and the frequency.

When the reactance is required and the impedance is known, we must first find the **natural** resistance, using direct current, for $R = \dfrac{E}{I}$

Then $X = \sqrt{Z^2 - R^2}$.

Considering the same coil as before, connected to a direct e. m. f. of 6 volts, we find with an ammeter that 2 amperes are flowing. Then $R = \dfrac{6}{2} = 3$ ohms, and $X = \sqrt{220.02^2 - 3^2} = \sqrt{48408.8004 + -9} = 220$ ohms.

To obtain the **resistance** when the reactance and impedance are known, $R = \sqrt{Z^2 - X^2}$.

Considering the same coil as before:
$R = \sqrt{220.02^2 - 220^2} = \sqrt{48408.8004 + -48400.} = 3.$

When a current flows through a conductor **heat** is produced in it. This is due to the resistance of the conductor to the current traversing it. It might be supposed that the temperature of a conductor rises in direct proportion to the current; but, instead of doing

this it increases as the square of the current. For instance, if one ampere produces a certain (instantaneous) value of heat, two amperes will produce four times as much heat; three amperes, nine times as much, etc., the resistance being the same in each case. From this we see that the instantaneous rate of the development of heat is proportional to the **resistance times the instantaneous value of the current squared**; or, R I². This is called **Joule's law.**

This formula also represents the instantaneous **power** consumed in heat, since power $W = E I$ and as $E = R I$, we see that $W = R I \times I$, or $R I^2$. It might be supposed that in **alternating** currents the average value of the wave would be as effective as a direct current having a value equal to the average alternating current, but instead it will be seen that an alternating current is as effective as a direct current which is .707 times the maximum value of an a. c. wave.

The average value of e. m. f. we found to be $\frac{2}{\pi}$ or .636 times the maximum value, and as the current consumed in resistance is in phase with the e. m. f. the average value of the current wave is .636 times $I_{max.}$ and the effective value is .707 times $I_{max.}$

To obtain the effective value, the current is squared at every instant from zero to maximum, but for most practical purposes the instantaneous values for several successive equal intervals of time are squared.

These squares are then added together and the average or mean square is found by dividing the sum by the number of intervals of time chosen. The square root of this average, or mean square, gives .707 of the maximum value. To get very accurate results requires the use of higher mathematics.

Most voltmeters and ammeters give directly the

effective values of the e. m. f. and current respectively, so to obtain the maximum value whenever required, the effective value must be multiplied by $\dfrac{1}{.707}$ or 1.414.

If, for instance, the effective e. m. f. E is 110 volts, then $E_{max.}=$ E $\times$ 1.414, or 110 $\times$ 1.414 $=$ 155.4 volts. The maximum value is not used as often as the effective value, but in this treatise recourse will often be made to it.

CHAPTER XI.

Eddy Currents.

The magnetic circuit of alternating-current machinery must not be made from solid iron or steel, otherwise excessive and wasteful eddy currents will be produced which may heat the iron to a temperature which might be harmful to the windings.

Eddy currents can not be entirely eliminated, but may be so reduced as to be negligible by subdividing the iron or steel longitudinally. In induction coils this is accomplished by using a bundle of soft-iron wires for the core. In transformers, generators and motors, the subdividing is accomplished by using soft sheet iron or steel. The thinner the sheets, or the smaller the iron wire, the less will be the eddy currents.

These currents are induced in the same manner as currents are produced in a secondary coil. The solid core can be considered as a conductor of one turn of very low resistance. Although the e. m. f. generated in one turn is very low, yet very heavy currents may be produced in the core in consequence of its low resistance. It is to increase this resistance that the core is laminated. The paths of the eddy currents in a solid core are shown in Fig. 71 by the small arrows. The magnetic flux is, of course, lengthwise through the core.

In Fig. 72 is shown how the laminations break up the circular paths of the eddy currents, the oxidation of the iron generally being sufficient to keep the current from flowing from sheet to sheet. Eddy currents

Fig. 7.1.

Fig. 72.

10
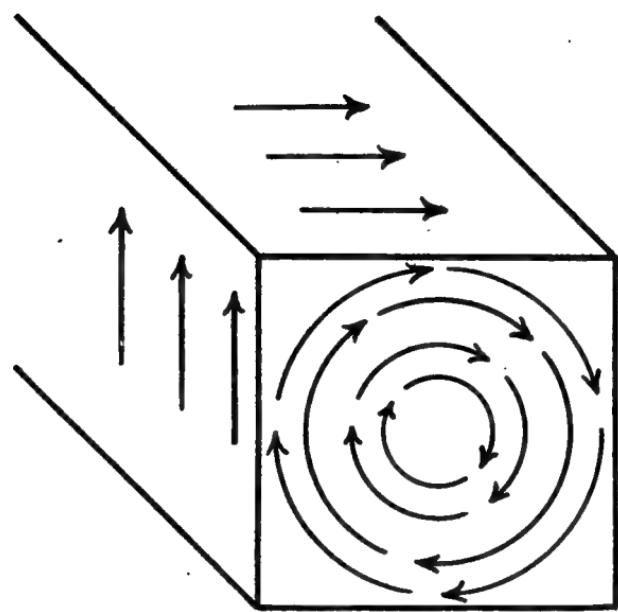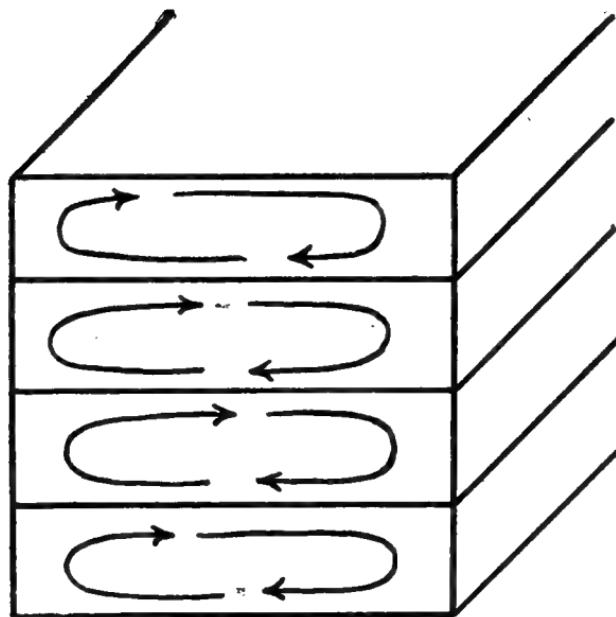
are produced, no matter how thin the iron, but become insignificant when the laminations are very thin.

Hysteresis.

Another serious loss in the magnetic circuit is hysteresis. The loss, as well as the eddy-current loss, raises the temperature of the iron, the heating being caused by the internal friction of the whirling iron molecules every time the magnetic flux changes its direction. The higher the frequency and the greater the flux density in the iron, the greater will be the hysteretic loss.

To keep the hysteresis loss low, it is necessary to use the softest iron or steel, and keep the flux density as low as practicable, which results in the use of larger cores.

Because of the retentivity of the magnetism in iron and steel the flux lags behind the magnetizing current. The changes in flux density during one and one-quarter cycle are shown graphically in Fig. 73, in what is called a loop of hysteresis.

$+$ I S represents the magnetomotive force when the current flows through the magnetizing coil in one direction, and $-$ I S when the current flows in the opposite direction. The m. m. f. $+$ I S produces the flux $+ \Phi$ from o to a when the magnetizing current increases from zero to maximum, and the magnetism falls from a to b as the current falls to zero. The residual magnetism is from o to b, and it requires the m. m. f. $-$ I S from c to o to bring the magnetism from b to zero at c. The m. m. f. required to demagnetize the iron of its residual magnetism is called the coercive force.

The m. m. f. $-$ I S from c to maximum creates the flux $- \Phi$ from c to d, and this flux falls to e when $-$ I S falls to zero. It requires the m. m. f. $+$ I S from o to f to bring the magnetism from e to zero at f. The
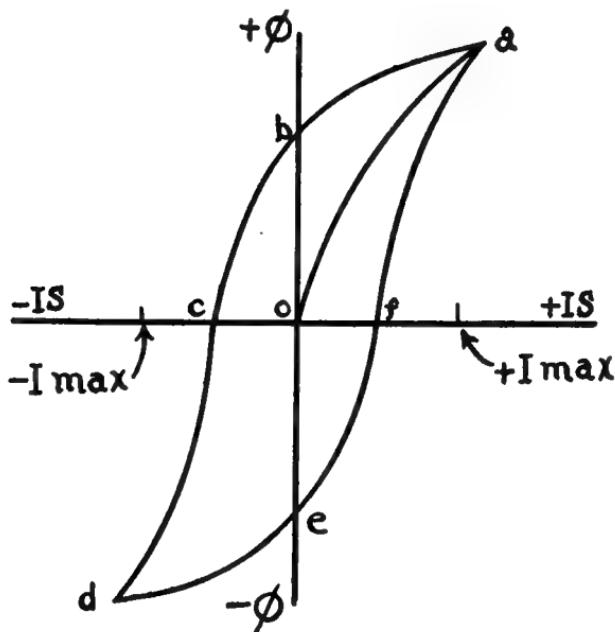
FIG. 7:3.

m. m. f. from f to maximum creates the flux from f to
a. The flux never again increases as it did from o to a,
because of the residual magnetism in the iron. Every
succeeding cycle of flux changes from a to b, c, d, e, f
and a.

When the residual magnetism is great the loop
becomes wide, and shows that the iron or steel tested
is unsuited for cores, but when the loop is narrow it
shows that the iron or steel is very good.

In order to understand the following subject, **Trans-
formers,** it was necessary to first understand the mag-
netic properties of iron and steel, and especially the
magnetic circuit.

Transformers.

As alluded to before, a transformer consists chiefly of three parts, namely, the primary winding, the magnetic circuit and the secondary winding.

The only connecting link between the primary and secondary windings is the magnetic circuit, which is insulated from both windings, as shown in Fig. 74.
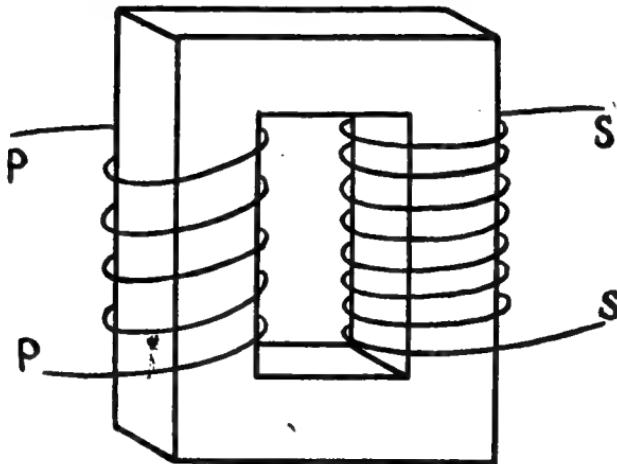
FIG. 74.

It is only for convenience that in Fig. 74 the primary and secondary windings are placed on separate legs of the core, for in most transformers the one winding is wound right over the other to prevent magnetic leakage. As practically all the flux created by the primary passes through the iron, or steel, core, the most of the lines interlink with the secondary winding also.

The ever-changing flux produced by an alternating

current in the primary threads through the secondary
and thus **induces** in it an alternating e. m. f.

Because of the self-induction in the primary, the
impressed e. m. f. is balanced by the counter e. m. f.
in its turns. Thus, the changing flux cutting the pri-
mary turns generates an e. m. f. **equal** to the impressed
voltage. It is then evident that if the same flux
threads through a secondary coil having the same num-
ber of turns as the primary, the induced voltage will
be the same as the impressed e. m. f.

If, for example, the effective voltage at the ter-
minal of the primary is 100, then the counter e. m. f.
must also be 100 volts. If the number of turns in the
primary is 200, the flux in moving in and out of 200
turns generates 100 volts, or ½ volt per turn. If there
are 300 turns in the secondary, the voltage is 300 times
½ volt, or 150 volts. Should the number of turns be
20,000 in the secondary, the voltage would be 20,000
times ½ volt, or 10,000 volts.

From these facts the relation of voltage and turns
of primary and secondary are expressed by the ratio:

<div align="center">

Secondary turns are to primary turns

as

Secondary voltage is to primary voltage.

</div>

If, for example, one has a transformer wound for
110 volts on the primary side, and it has 330 turns;
and he should wish to know how many turns are
needed to give 11,000 volts, the required number of
turns can be found by substituting the different values
in the formula, thus:

110 : 11,000 :: 330 : number of turns required.

$$\frac{110}{11,000} = \frac{330}{33,000} \quad \therefore \quad 33,000 \text{ turns are required.}$$

When the secondary coil is on open circuit the
effect upon the primary winding and the magnetic
circuit is as though the secondary were not present.

However, when current is taken from the secondary, this current in passing through the secondary tends to produce flux in the opposite direction from that of the primary, thus weakening the original flux. Weakening the flux lowers the self-induction in the primary winding, consequently the voltage impressed upon its terminals forces more current in the primary to keep the flux normal. We see from this that the flux that interlinks both windings is practically the same at no load and full load.

When the secondary is on an open circuit the current flowing in the primary consists of the wattless magnetizing current and that required to supply the core losses.

Raising the voltage impressed on the primary raises the current, and, consequently, the flux density;
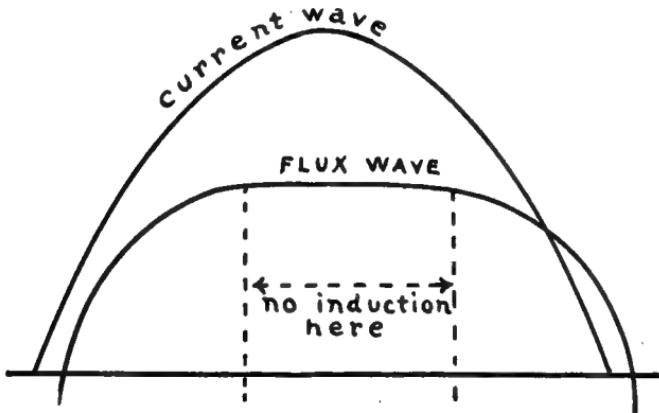
FIG. 75.

and if the voltage gets too high the core becomes saturated before the current in the primary has reached its maximum. The result is that the top of the flux wave becomes perfectly flat, as shown in Fig. 75.

The surplus current is wasted in heating the primary coil, as no inductive effect is produced in the secondary by the flat part of the wave. It is better, therefore, to use a lower voltage on the primary when a high efficiency is desired.

There are two main designs of transformers, called, respectively, the **shell** (Fig. 76), and the **core** types (Fig. 74). Each possesses certain advantages over the other.
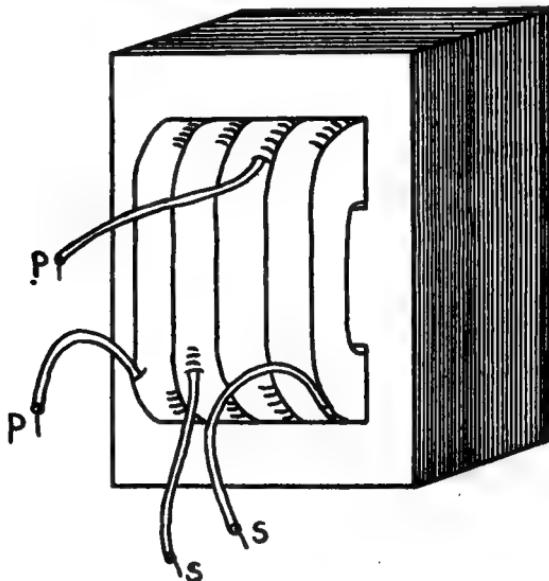
FIG. 76.

A newer type is shown in Fig. 77, which is really a combination of the other two designs. The aim is to get a magnetic circuit of as low reluctance as possible and as large a cooling surface on the coils as can be obtained, combined with good regulation.

END VIEW
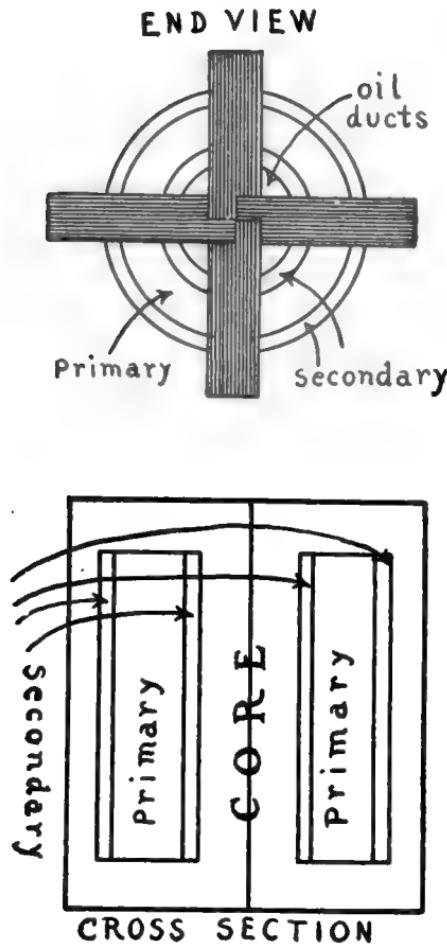
FIG. 77.

The **radiation** of heat into the surrounding air may
be sufficient to keep a small transformer cool, but

might be entirely inadequate to keep a large one from becoming dangerously hot. The reason for this is that the surface of a transformer does not increase in proportion to its size. It therefore becomes necessary to use artificial means of increasing the surface by making ducts vertically through the transformer. In some large transformers a blast of air is blown through the ducts, but those of medium size are filled with oil, which circulates freely through the core and windings. In some very large transformers the circulation is assisted by a pump, while in others the oil is cooled by cold water, which is forced through a coil of pipe placed in the transformer case.

Running a transformer at a high temperature not only endangers the insulation of the windings, but also deteriorates the iron or steel in the core. This deterioration is called ageing.

Ageing takes place in iron after long service, even though the temperature has not been excessive, but heat increases it. Some steels which have good non-ageing qualities are now given the preference over those steels of high permeability which age quickly. Steel alloyed with silicon has good nonageing qualities.

The oil used in transformers serves the twofold purpose of a cooling medium and a liquid insulation. Oils for this purpose should be quite thin and have a high flashing point, should be as nonhydroscopic as possible and should have a high insulating strength.

The construction of a core-type transformer of high voltage, such as will be used in subsequent experiments, is here given:

The core is made from thin sheets of soft steel into a rectangle 8 by 14 inches outside, and 3 by 9 inches inside measurements. The cross-section is 2½ inches square. The strips composing the core are of two lengths, namely, 5½ inches and 11½ inches, but both sizes are of the same width, 2½ inches.

The long strips, in groups of three or four at a time, are made into two cores like the one shown in Fig. 78.

FIG. 78.

These cores should each be clamped in a vise and the middle 9 inches of length wrapped tightly and evenly with three layers of friction tape, over which should be wrapped two layers of Empire insulating cloth. A little shellac will hold the cloth in place.

Four 60-foot lengths of No. 13 double cotton-covered wires are used in the primary. Two of these wires are used at a time and wound side by side in a single layer of fifty-five double turns over the empire cloth. The wire is wound tightly and evenly, care being taken that the cotton insulation is not injured. The ends of the wire are left about a foot long for connections. About four pounds of wire are used.

The two cores are now connected together at one end by the 5½-inch strips, making a rectangle with one side unfinished, as shown in Fig. 79.

The bottom of the core is held in place by two boards, one on each side, drawn tight by bolts.

The four lower ends of the primary coils are connected and soldered together so that they become two distinct windings of 110 turns each. See Fig. 79. These are connected in series by switching 2 and 3,
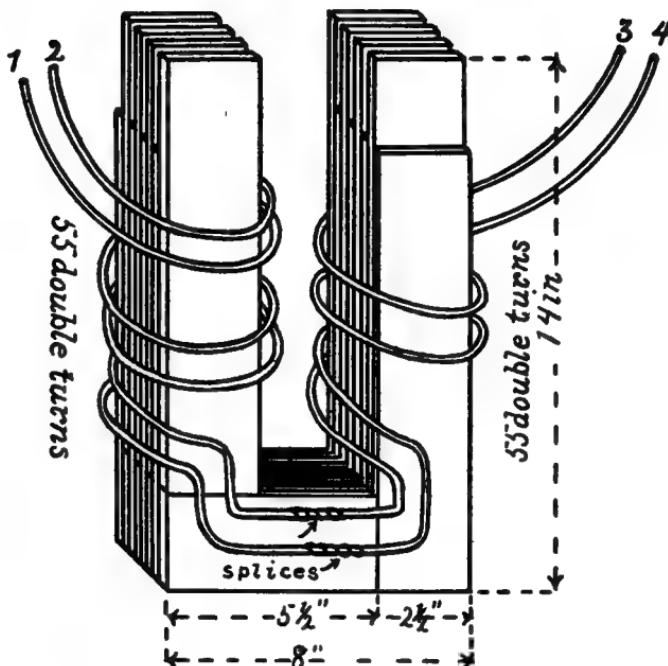
FIG. 79.

using 1 and 4 for terminals. For parallel connection 1 and 2 are switched together as one terminal, while 3 and 4 are switched together for the other terminal. The upper ends of the wire should be left standing straight up so that the secondary coils may be easily slipped down over the primary winding without injury.

Fifteen layers of No. 7 empire cloth, nine inches wide, are wound over the primary on each leg of the core. Built up mica tubes are still better, but rather expensive. This constitutes the major insulation between the primary and secondary.

The secondary is wound in eight sections, each

having 2,500 turns of No. 32 single cotton-covered wire. There are about 6½ pounds of wire in the secondary. The sections are six inches square outside and four inches square inside dimensions, and 1¼ inches wide. They are wound on a wooden form almost four inches square and 1¼ inches wide. The edges across the width of the block are rounded off. The block should be perfectly smooth and straight so that the completed coils can be taken off easily. This block is centered on a metal rod of convenient size and about 6 or 7 inches long. On one end is fitted a small crank if the winding is done by hand. To keep the coil from sliding out of shape, two metal (or wood) discs about 8 inches in diameter are screwed to the sides of the form (Fig. 80).

The spool of No. 32 wire is mounted so as to revolve easily and placed about 5 or 6 feet away from the form. A smooth metal or glass rod is set about a foot away from the form. After mounting the form between two supports, it is turned by the crank with the right hand while the wire is guided by the left hand as it slides over the rod.

As heretofore explained, the end of the wire is first passed through a small hole in one of the metal discs and a strip of empire insulating paper 1¼ inches wide wrapped around the form once, letting the ends overlap about an inch. The wire is then wound in layers back and forth until 2,500 turns have been wound. The layers are separated by layers of empire paper, letting the ends overlap a little. A margin of ⅛ inch at the edge of the paper is necessary to keep the wire from slipping off.

The last end is fastened by wrapping three or four turns of white sewing thread around the outside layer and tying the end of the wire to this. The coil is taken off the form by unscrewing one of the discs and carefully pulling out the first layer of wire.

The coils are frequently tested for open circuits
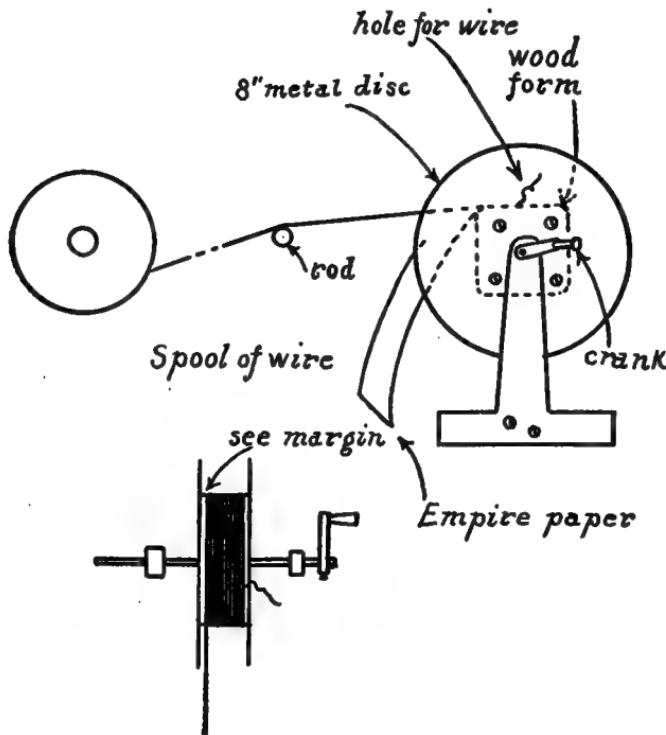
FIG. 80.

while they are being wound. A short circuit will often cause a burnout but an open circuit is even worse. Do not attempt to use a coil in which there is either a short or open circuit. All joints should be soldered, but rosin instead of acid should be used as a flux. If any large bare places are found on the wire, the entire bare piece should be cut out and the ends spliced and soldered neatly and smoothly. A thin piece of silk folded over the joint once is a sufficient covering and

at the same time it does not make a lumpy place as the adjacent turns are wound over the flaps to hold them down.

The spools of wire, before winding the coils, should be thoroughly dried out, in a warm oven not too hot. The hands of the person who winds the coils must be clean and dry.

After the eight coils have been completed they should be dried out in a medium warm oven, immediately after which they should be thoroughly impregnated with insulating compound. To do this, the coils should be laid flatwise in a large flat-bottomed pan full of melted extra amber petrolatum (vaseline). The mixture should simmer for several hours, care being taken that the coils do not burn on the bottom. When the coils have been completely saturated, the petrolatum should be allowed to cool until set, which takes several hours.

When thoroughly cold the coils should be taken out, the surplus petrolatum wiped off and the coils again tested for open circuits, after which the coils can be assembled on the core over the primary winding. The lower coils should rest on a sheet of micanite 6 by 12 by 1/10 inches, which fits over the legs of the core and primary and rests on the binding boards around the bottom of the core. The mica sheet should be raised high enough to make a clearance of about 1 inch between the lower coils and the core.

Adjacent coils should be separated by six or more thicknesses of empire cloth. The cloth is cut in 7-inch squares, with a hole in each, which fits closely over the primary winding and insulation. Another sheet of micanite, 9 by 4 by 1/10 inches, is used to separate the two vertical sets of coils.

The proper way to connect the coils together is shown in Fig. 81. By beginning all coils on the same side of the form when winding, uniformity of connec-

tions is obtained. If this is done, the inside end of the first coil on top on one leg may be connected to the inner end of the second coil, and the outside end of the second to the outside end of the third, and so on, until only the first and last ends are left open. The object in connecting the coils in this way is to insure good insulation and the proper relation as to the direction of turns in the coils.

All joints should be soldered. This can be done with the inner joints very easily if the coils are taken in pairs before assembling on the core, as then the soldering copper can be inserted into the square hole in the coils. The inner connections should be made no longer than necessary, for they might reach another part of the coil, and in so doing might cause a short circuit.

After all connections have been made, the secondary should again be tested for open circuits. If everything is all right the upper portion of the core may be completed, and the whole transformer set vertically in a wooden box 7 inches wide, 14 inches long and 15 inches high inside (Fig. 81).

The box may be screwed fast to the transformer by means of the binding-boards at the bottom of the core. Keeping the fine-wire ends of the secondary 6 inches apart, the box should then be filled to the top with very hot petrolatum. About 40 pounds are required in all.

The primary winding is controlled by a single-pole, double-throw knife switch, as shown in Fig. 82. The ends 2 and 3 of the primary are soldered together and then connected to one jaw of the switch, while number 4 is connected to the other jaw.

The line wires from the mains are connected to L_1 and L_2. When the blade is thrown into jaw 4 the primary windings are in series, but when thrown into

the other jaw only half the number of turns in the primary are used.

To vary the strength of current in the primary, the reactance coil in Fig. 69 is used in series with the line current. As the pressure of secondary runs up to much more than 20,000 volts at times, the fine-wire ends of the secondary should be carried up through the cover of the box, through thick-walled tubes to binding-screws at the top of the tubes or posts. The tubes may be made from hard rubber, porcelain, micanite or glass.

The greatest practical application of the class of transformers such as described is high-voltage transmission of current. The advantage of a high-voltage line is the economy of copper in transmission of power, for a small current at a high voltage will produce as much power as a large current at a low voltage.
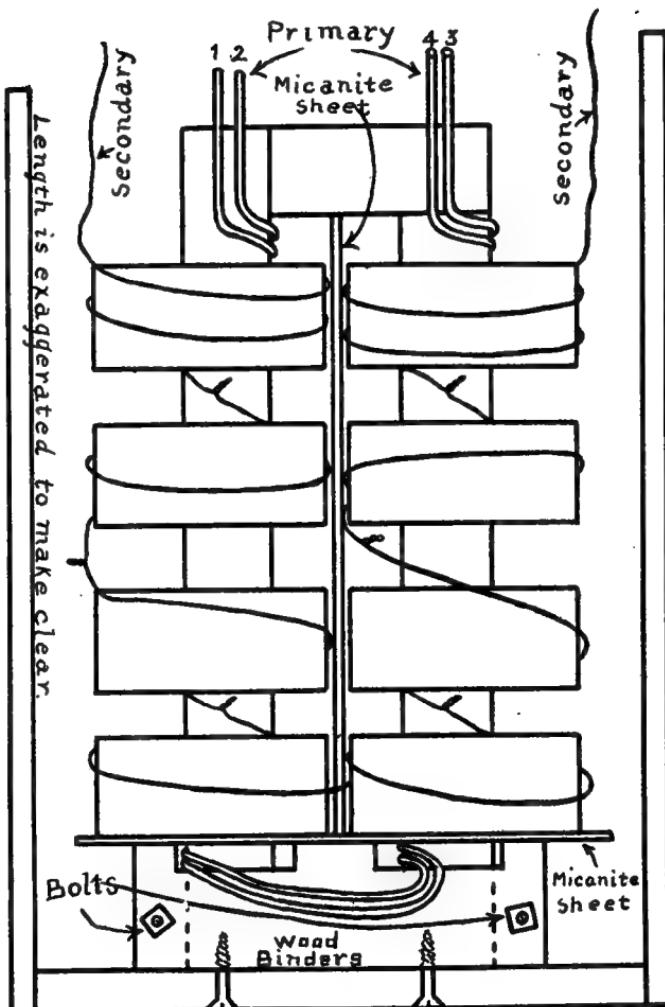
FIG. 81.

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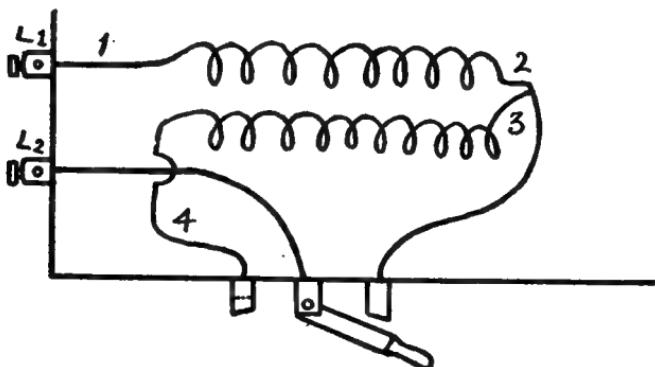
FIG. 82.

The line wires of a low-voltage system must be very thick to have low resistance and to carry the large currents required, consequently the cost of a low-voltage transmission line of great length would be prohibitive.

At the present time some companies are successfully operating transmission lines at over 100,000 volts. By means of step-up transformers at the generating station, the voltage is raised as high as desired, and sent at this pressure to the receiving substation, where the voltage is reduced by a step-down transformer to the pressure of the local distributing lines. In this way a system is obtained which is both flexible and economical.

Difficulties present themselves in high-voltage transmission which are negligible in low-voltage lines. Chief of these is line insulation. Ordinary small glass insulators would be useless on such lines; instead, insulators are made which are a foot or more across and weigh between 30 and 40 pounds each. Several insulators are sometimes linked together and hung

from below the crossarm, the wire being suspended from the lowest insulator. Even with such insulators as these, the leakage is sometimes considerable.

Because of the great length of transmission lines, the **self-induction** becomes an important factor, besides which another factor is introduced, called **capacity.**

CHAPTER XII.

The **capacity** effect in a circuit is due to the peculiar actions of static electricity.   Static electricity can be produced by friction.   If a glass rod is rubbed with silk, both the rod and the silk acquire the new property of attracting small, light bodies, such as cork dust, small pieces of paper, etc., in much the same manner as a magnet attracts iron filings.   The glass rod and silk are not magnetic, however, for a compass needle is unaffected by their presence.   The phenomenon of the attraction for the small, light bodies by the rod and silk is due to their electrification, the former being charged with positive electricity and the latter with negative electricity.

When a stick of sealing wax is rubbed with flannel the wax becomes negatively charged and the flannel positively charged.   These, also, have the property of attracting small, light bodies.   In fact, all substances may be electrified under appropriate conditions.

If the electrified glass rod be suspended by a thread and the electrified stick of sealing wax be brought near the suspended rod, they will be attracted to each other. A positively charged glass rod brought near to the suspended rod will repel it.

From this we obtain the law that charges of **opposite** signs attract each other and charges of **like** signs repel.   This law is similar to the law of magnet poles, in which unlike poles attract, and like poles repel each other;   yet the repulsion and attraction of magnet poles are due to an entirely different cause from that of electric attraction and repulsion.   In both cases the

attraction and repulsion are due to a displacement of the ether, but the distinction between magnetic and electric phenomena lies in the nature of the displacement. As the lines of magnetic force show the displacement of ether in magnetism, so there are also electric lines of force which show the displacement of the ether in and around charged bodies.

When two oppositely charged bodies are brought near each other the electric lines of force assume the position in Fig. 83, but when two similarly charged bodies are brought near to each other the lines take the form in Fig. 84. There is a striking resemblance between the fields of electric and magnetic forces.

By convention, the electric lines of force are said to move from the positive charge to the negative, just as the magnetic lines of force are said to move from the north pole to the south pole.

When the charged body is a nonconductor, the charge is local; that is, the charge remains only on the part that has been rubbed. With charged bodies of metal or other conductors the case is different — the charge immediately distributes itself over the entire body. As every part of each charge is of the same sign, every part is repelled by every other part; thus the tendency of every part is to get as far apart from the rest as possible. As the farthest they can get apart is at the surface, the charge resides on the surface of the body.

An instrument used to detect electric charges is called an **electroscope.** One form consists of two small strips of gold leaf, suspended from a metal rod in a glass jar or bottle. When a charge is imparted to the rod, and consequently the gold leaves, the strips diverge, either slightly or widely, depending upon the potential of the charge. The leaves diverge because they are both charged alike, and thus repel each other whether it be a positive or negative charge.

If a positively charged metal ball be suspended by a silk thread and allowed to touch the rod of the electroscope, the leaves will diverge.   If an uncharged
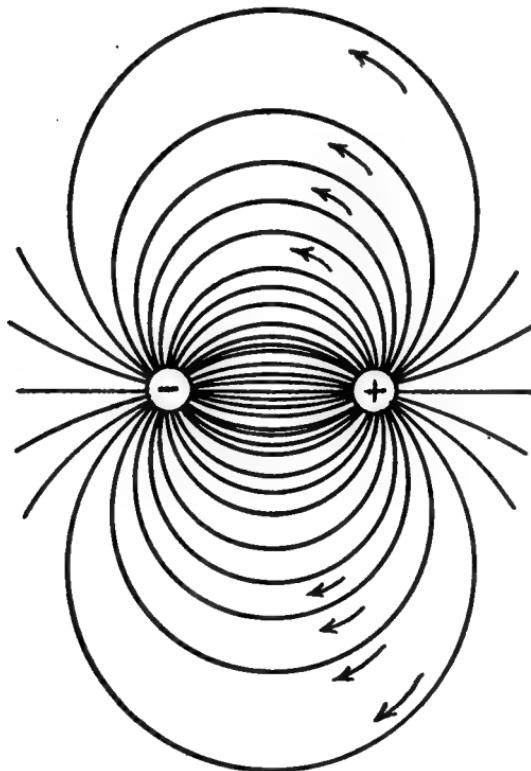

Fig. 83.

metal ball (or piece of metal), also suspended by a thread, be allowed to touch the charged metal ball, the leaves of the electroscope will not diverge as

widely as before. This is because the charged ball gave part of its charge to the other ball, which caused a fall in potential. In analogy, let a tank filled with air at a high pressure represent the charged metal ball, and let an empty tank represent the uncharged ball. Connecting the two tanks together, so as to allow the air under high pressure to fill the empty tank also, illustrates the transfer of a quantity of electricity from the charged ball to a ball which was not charged.
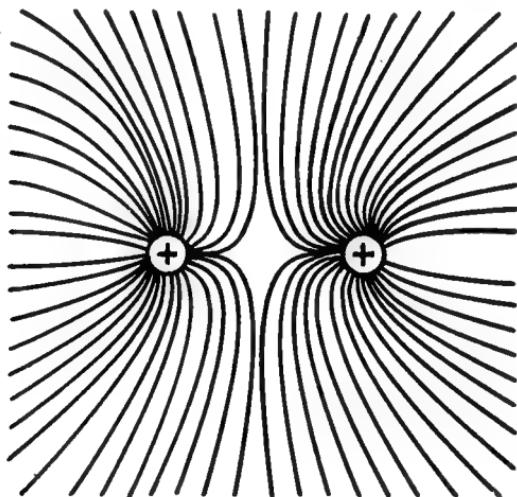

FIG. 84.

If we compare the quantity of air in the tank to the quantity of electricity on the charged ball, and the pressure of the confined air to the potential of the electric charge, it is evident that as the pressure of the air when occupying two tanks is lower than when occupying one, so, likewise, the potential of the charge

distributed on two balls is lower than when the entire charge is confined to one ball.

The density of the lines of electric force is greater when the charge is confined to one ball than with two, because the area over which the lines are distributed is greater in the latter case. This compares with the increase in cross section of a magnetic circuit, reducing the density of the magnetism per unit area.

If instead of using a metal ball, a metal plate is used, the potential of the charge is lowered still more than before, in virtue of the increased area over which the lines are distributed.

Now, if a negatively charged plate is brought near the positively charged plate which we have been considering, the potential is again lowered, as indicated by a partial collapse of the leaves of the electroscope. Bringing the plates closer to each other shortens the path of the electric lines of force, and consequently increasing their number in a way analogous to the increase in the number of magnetic lines due to shortening the magnetic circuit.

Should a plate of glass be inserted between the charged plates, the leaves of the electroscope might collapse almost completely. The reason for this is that the glass presents an easier path for the electric lines of force than the air does, just as the iron presents an easier path for magnetism than air does, due to the higher permeability of the iron. So glass has a higher permeability to electric lines of force than air, but this quality is not termed permeability in the case of electric lines of force, instead it is called the dielectric constant or dielectric capacity of the glass.

The medium between the two charged conductors, through which the electric flux must pass, is termed the **dielectric**. The ratio between the dielectric capacity of a substance to that of air is called the specific inductive capacity of the substance.

The approximate values of the specific inductive capacity of a few substances follow, namely:

Air ....1.	Mica ....4.6 to 7.98
Vacuum .... .9996	Paraffin ....1.85 to 2.32
Castor oil 4.7	Porcelain 4.38
Petroleum ....2.	Rosin ......2.48 to 2.57
Sperm oil.........3.	Rubber (vulcan-
Turpentine 2.2	ized) 2.69
Vaseline 2.17	Shellac .....2.74 to 3.73
Ebonite .....1.9 to 3.48	Wax 1.86
Glass ....2.5 to 9.	

Instead of charging the conductors by friction (the method used in the foregoing experiments), the charging may be accomplished by connecting the conductors to some electric machine or battery. Assume that a battery is used: The conductor connected to the carbon becomes positively charged, while the conductor connected to the zinc becomes negatively charged. Although the two conductors are separated by the dielectric, a momentary current flows from the battery into the conductors. This current flows only long enough to charge the conductors to a difference of potential equal to that of the battery. Thus a certain quantity of electricity is stored up in the conductors and dielectric in the form of electric flux.

By virtue of the storing-up qualities of two conductors separated by a dielectric, this arrangement has been termed a condenser, although the name is not entirely appropriate.

The larger the plates, the closer they are together, and the larger the inductive capacity of the dielectric, the greater is the capacity of a condenser to store up energy in the form of electric flux. Condensers are sometimes called capacities, but this term is often limited to certain applications.

To obtain condensers of large capacity, a number

of plates or sheets of metal are used instead of two single plates or balls. Tin or lead foil are the metals generally used in the construction of condensers. Several sheets of foil are laid flatwise in a stack alternately with sheets of mica, glass or paper, much larger than the sheets of foil. Then all the even-numbered sheets of foil are connected together to form one terminal of the condenser, while all the odd-numbered sheets are connected together to form the other terminal (Fig. 85).

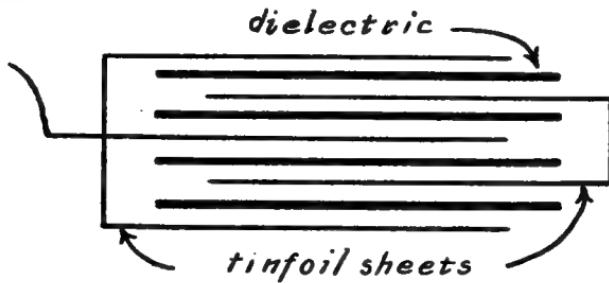*dielectric*

tinfoil sheets

Fig. 85.

The quantity of electricity that can be stored up in a condenser is also determined by the potential of the charge that it can stand. In magnetism a point of saturation is reached where the iron can not be forced to carry more flux, but there is apparently no such limitation to the flux density in the dielectric of a condenser. There is, however, a limit of potential, which, if exceeded, will puncture the dielectric and discharge the condenser. If the dielectric is a gas or oil the puncture heals itself, but when a solid is used, such as glass, the dielectric is destroyed by the puncture.

The relation between the capacity, quantity and potential is expressed in the equation: $C = \dfrac{Q}{E}$

When the quantity of electricity is expressed in coulombs, and the potential in volts, the capacity is given in **farads**. By definition, a condenser whose capacity is 1 farad is one whose potential will be raised from zero to 1 volt when charged with 1 coulomb of electricity.

From the preceding equation we deduce that the coulombs of electricity in a condenser of C farads, charged to a potential of E volts $=$ CE; and we also find that $E = \dfrac{Q}{C}$

The **farad** is much too large a unit for practical use, so a smaller unit, called the **microfarad,** is used. Its value is one-millionth part of a farad.

The electromagnetic, or absolute unit of capacity, is even larger than the farad. It is defined as the capacity of a condenser which will be charged to unit potential by a unit quantity of electricity. As unit current is 10 amperes, and unit quantity is unit current flowing for one second, it is obvious that the absolute unit of quantity is 10 coulombs.

Unit potential being $\dfrac{1}{100,000,000}$ volt, and as $C = \dfrac{Q}{E}$ a condenser of unit capacity equals $\dfrac{10 \text{ coulombs}}{\dfrac{1}{100,000,000} \text{volt}} =$

1,000,000,000 farads.

Conversely, a farad equals $\dfrac{1}{1,000,000,000}$ absolute unit of capacity.

In another system (the electrostatic) the unit of capacity is also defined as that of a condenser whose potential is raised to unity by unit quantity of electricity. The units of potential and quantity, however,

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are different in this system from those of the electromagnetic.

The unit of work in both systems is the same, but the velocity of a conductor in the electrostatic system is the speed of light, or approximately 30,000,000,000 cms. per second as compared to 1 centimeter per second in the electromagnetic system.

As stated before, a force of 1 dyne is exerted upon a conductor by a unit field when the current in the conductor is 10 amperes. The conductor moved against this force through 1 centimeter performs 1 erg of work.

If this conductor is moved against the same force through 30,000,000,000 cm., it would perform 30,000,-000,000 ergs of work; but, if the current in the conductor is reduced to $\dfrac{1}{30,000,000,000}$ of 10 amperes, the work performed by the conductor would be only $\dfrac{1}{30,000,000,000}$ as much, or 1 erg. Therefore, the current required in a conductor which performs 1 erg of work in moving 30,000,000,000 cm. across unit field is $\dfrac{1}{30,000,000,000}$ of 10 amperes, or $\dfrac{1}{3,000,000,000}$ ampere.

This is the value of unit current in the electrostatic system. Unit current for one second equals unit quantity in this system, or $\dfrac{1}{3,000,000,000}$ coulomb.

The conductor in the electrostatic system moves across unit field at a uniform velocity of 30,000,000,000 cm. per second, consequently generating an e. m. f. of 300 volts, which is unit of e. m. f. in this system.

Now, a condenser of unit capacity in this same system is one which $\dfrac{1}{3,000,000,000}$ coulomb will raise from

zero to a potential of 300 volts. This capacity is obtained in farads by the equation:

$$C = \frac{Q}{E}, \text{ or } \frac{\frac{1}{3,000,000,000} \text{ coulomb}}{300 \text{ volts}} = \frac{1}{900,000,000,000}$$

farad. This can also be expressed as 9×10^{-11} farad.

Conversely, a farad is equal to 9×10^{11} electrostatic units of capacity.

The capacity of isolated spheres are found to vary as their radii. A sphere having a radius of 1 centimeter, hung up in space at an infinite distance from any other conductor, has unit electrostatic capacity; so the capacity of spheres in electrostatic units can be given directly from their radii in centimeters.

Consequently, a sphere having a radius of 9×10^{11} cm. has a capacity of 1 farad, and a sphere of 900,000 cm. radius equals 1 microfarad; hence some authors write 1 microfarad as 900,000 cm. of capacity.

The work done in charging a condenser can be determined by the time it takes a certain e. m. f. to charge a condenser of known capacity to the potential of the impressed e. m. f. Assume that the condenser is being charged at a uniform rate; that is, that the number of coulombs flowing into the condenser is the same for every second of time required to charge it to potential E.

As the total number of coulombs in the condenser equals C E, for $Q = C E$, then the number of coulombs flowing per second is the total charge, or C E, divided by the number of seconds T it takes to charge the condenser, or $\frac{C E}{T}$. The quantity $\frac{C E}{T}$ represents the mean number of amperes flowing, for a coulomb of electricity per second requires a flow of 1 ampere.

Because the condenser is being charged uniformly, the potential rises uniformly from zero to its final value, consequently the mean value of the potential is just half of the final value, or $\frac{E}{2}$, as shown in Fig. 86.
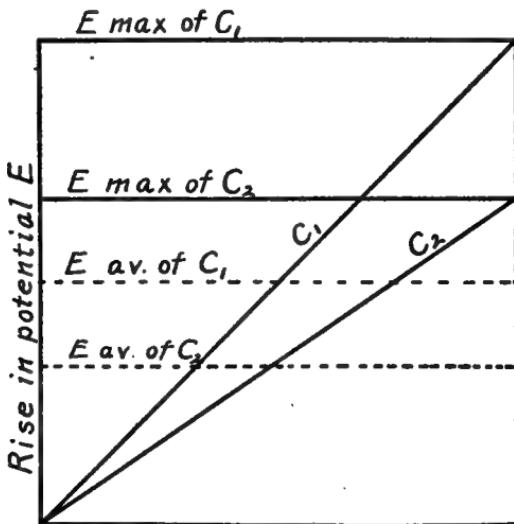

FIG. 86.

The product of the mean number of amperes by the mean number of volts gives the mean number of watts, or the rate which the charge is being stored up, for $\frac{C E}{T} \times \frac{E}{2} = \frac{C E^2}{2 T}$ watts.

It takes T seconds to charge the condenser; therefore, $\frac{C E^2}{2 T}$ watts are active for T seconds and the energy stored up equals $T \times \frac{C E^2}{2 T} = \frac{C E^2}{2}$ watt seconds or joules.

By inspection of the last member of the equation, it is evident that the time required to charge a condenser does not affect the amount of energy stored up; that is, as much energy is stored up at a certain potential, whether the condenser is charged slowly or quickly.

In a condenser of 1 farad capacity, charged to a potential of 10 volts, there are stored up $\dfrac{1 \times 10^2}{2} =$ 50 joules, or the same amount of energy as stored up in magnetism in a circuit of 1 henry inductance when 10 amperes of current are flowing through it.

As more energy is stored up in an inductive circuit by increasing the current until the magnetic circuit is saturated, so, also, more energy is stored up in a condenser by raising the potential of the charge. As stated before, a limit is reached which, if exceeded, will result in a **puncture** of the dielectric, yet it is possible with some forms of small-capacity, high-voltage condensers, to store up as many coulombs in them as a low-potential condenser of large capacity.

A condenser of ¼ m. m. f. capacity, charged to a potential of 20,000 volts (which is very common), represents as much energy as a farad condenser charged to 10 volts, for

$$\frac{.00000025 \times 20,000 \times 20,000}{2} = 50 \text{ joules.}$$

In the actual charging of a condenser, the rise in potential is not uniform, as assumed in the preceding discussion. When charged through a resistance having no self-induction, the rise in potential, at any instant after contact, can be determined by the following

formula: $e = E \left[1 - \epsilon^{-\frac{t}{CR}} \right]$

The small e represents the potential of the con-

denser t seconds after contact. The large E stands for the impressed e. m. f. and also the final potential of the condenser. The potential of the condenser at every instant acts as a counter e. m. f., so that instead of the full value of E being able to act, only a fraction is active. This fraction is equal to $E\epsilon^{-\frac{t}{CR}}$, and the potential of the condenser is the difference between this fraction and the impressed e. m. f., or $e = E - E\epsilon^{-\frac{t}{CR}}$.

In a circuit having a capacity of 3 farads, an e. m. f. of 8 volts and a resistance of 2 ohms, the rise in potential in two seconds is obtained by substituting these values in the equation:

$$e = 8 \left[1 - 2.718^{-\frac{2}{3 \times 2}} \right]$$

The exponent $-\frac{2}{3 \times 2}$ equals $-\frac{1}{3}$, so the equation can be written:

$$e = 8 \left[1 - 2.718^{-\frac{1}{3}} \right] = 8 \left[1 - \frac{1}{\sqrt[3]{2.718}} \right] =$$

$8 - \frac{8}{1.3956} = 2.26$ volts.

If the capacity is given in m. m. f., the resistance must be given in **megohms** (million ohms). The product of C and R is called the time constant of a condenser in the same manner that $\frac{L}{R}$ is the time con-

stant of an inductive circuit. When C R is large, a
long time is required to charge a given condenser;
thus the resistance in the charging circuit should be
kept as low as possible.

Consider a and b in Fig. 87 as the terminals of a
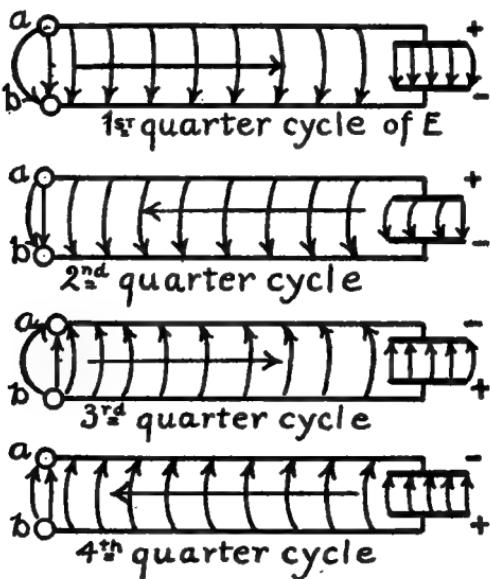transformer connected. to two parallel coatings of a

1st quarter cycle of E

2nd quarter cycle

3rd quarter cycle

4th quarter cycle

FIG. 87.

FLUX MOVES IN DIRECTION OF LONG ARROW.

condenser by two parallel conductors, such as used in
a transmission line. As the e. m. f. passes through
its cyclic changes, so, likewise, does the electric flux
between the transformer terminals. The two con-
ductors, however, offer a way of escape for the lines,
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which they are not slow to accept, but glide out along the conductors with the velocity of light.

Each electric line of force in a capacity is supported at its ends by two minute electric charges, a negative on one coating of the condenser, and a positive upon the other. These charges rush with and at the same speed as the lines of force, providing the conducting wires have perfect conductivity.

This transfer of electric charges along a conductor constitutes an electric current, and the greater the rate of transfer of these charges at any instant, the greater is the current at that instant.

The rate of change of the e. m. f. determines the rate of change of the electric flux; and as the rate of change of the e. m. f. is greatest as it passes through zero, so the rate of change of the electric flux is greatest when the e. m. f. is zero. A greater rate of change of flux produces a greater rate of transfer of electric charges or current, consequently the current is greatest when the e. m. f. is zero.

When the e. m. f. is maximum, the flux is stationary. No transfer of charges is taking place and no current flowing at that instant, so we see that the current is 90° ahead, or in time quadrature with the e. m. f., as shown in Fig. 88.

Because the current in a circuit containing capacity reaches its maximum before the e. m. f. reaches its highest value, the current is said to lead the e. m. f.

During the first quarter cycle the condenser coatings are charged to the potential of the impressed e. m. f.; but as the e. m. f. decreases during the second quarter cycle, the charges on the coatings, with the flux, rush back to the transformer. During the third quarter cycle the flux again rushes out, but the direction of the lines is reversed, as are also the signs of the charges. During the last quarter of the cycle the

flux and charges are again restored to the trans-
former, Fig. 87. This goes to show that the power
thus used in charging a capacity or condenser is watt-
less; that is, it only pulsates in and out of the circuit
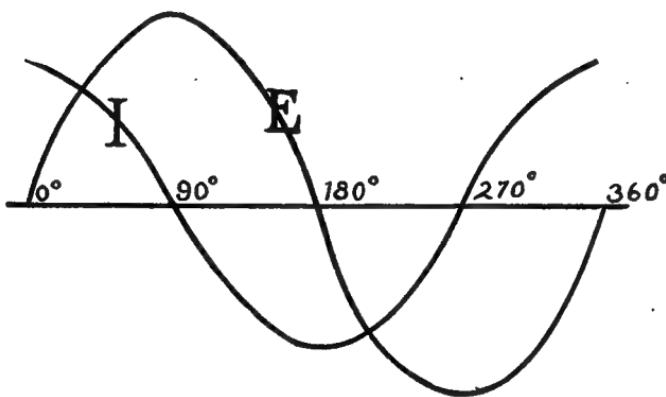without being used up.

FIG. 88.

The higher the **frequency** of the impressed e. m. f.,
the larger will be the current in a circuit containing
capacity, because the rate of change of flux is greater.
Increasing the capacity also tends to increase the cur-
rent, because an easier path is presented to the lines
of force, which increases the rate at which the charges
are transferred.

Although a capacity, or condenser, is an open cir-
cuit, their apparent conductivity is directly propor-
tional to their capacity and the frequency of the e. m. f.
Conversely, their apparent resistance, called capac-
ity reactance, is **inversely** proportional to their capac-

ity and the frequency of the e. m. f., and when the e. m. f. follows the sine wave the reactance is inversely proportional to $2 \pi f C$, or $X_c = \dfrac{1}{2 \pi f C}$ in ohms.

Since inductance causes the current to **lag** and capacity causes it to **lead**, inductance and capacity can be used to neutralize each other, as shown in the equation: $X = \sqrt{\left(2 \pi f L - \dfrac{1}{2 \pi f C} \right)^2}$, and when the circuit contains resistance which is not negligible the impedance is expressed as

$$Z = \sqrt{R^2 + \left(2 \pi f L - \dfrac{1}{2 \pi f C} \right)^2}\ \text{in ohms.}$$

Calling $2 \pi f L$ as X_s, and $\dfrac{1}{2 \pi f C}$ as X_c, the equation may be written: $Z = \sqrt{R^2 + (X_s - X_c)^2}$.

When the capacity reactance exactly equals the inductive reactance, the circuit is said to be in **resonance**, and the current in such a circuit can be obtained correctly by Ohm's law, $I = \dfrac{E}{R}$.

Since the difference between X_s and X_c is zero, when both reactances are equal, then $(X_s - X_c)^2 = 0^2$ or 0; so $Z = \sqrt{R^2}$, or simply R, as in a circuit containing only resistance.

CHAPTER XIII.

High-frequency Currents.

As stated before, the frequency of commercial alternating currents is seldom much more than a hundred cycles per second, but such frequencies are insignificant when compared with those now to be considered. Although no line can be drawn to distinguish where low frequencies leave off and high frequencies begin, yet it is customary to speak of frequencies expressed in thousands or hundred thousands as being high, while currents reckoned in millions and upward are generally called **oscillations.**

It is obvious that to construct a generator to produce a current of a frequency of even a few thousand cycles per second requires a very large number of magnet poles run at an exceedingly high speed.

The peripheral velocity of the revolving part of such a generator is enormous, sometimes as high as thirteen and one-half miles per minute.

Nikola Tesla, R. A. Fessenden and a few others have constructed some serviceable machines, but none have been able to obtain a frequency much over 100,-000. A Fessenden machine of this frequency is shown in Fig. 89. It is run by a 10 horse-power direct-current motor, although the rated capacity of the generator is only two kw. The full speed of the rotor of this machine is 20,000 r. p. m. This exceedingly high speed is accomplished by sets of gears which are in turn run by a chain-drive from the motor.

As a safety device the shaft, between the rotor and the gearing, is made very slender, so that the shaft is
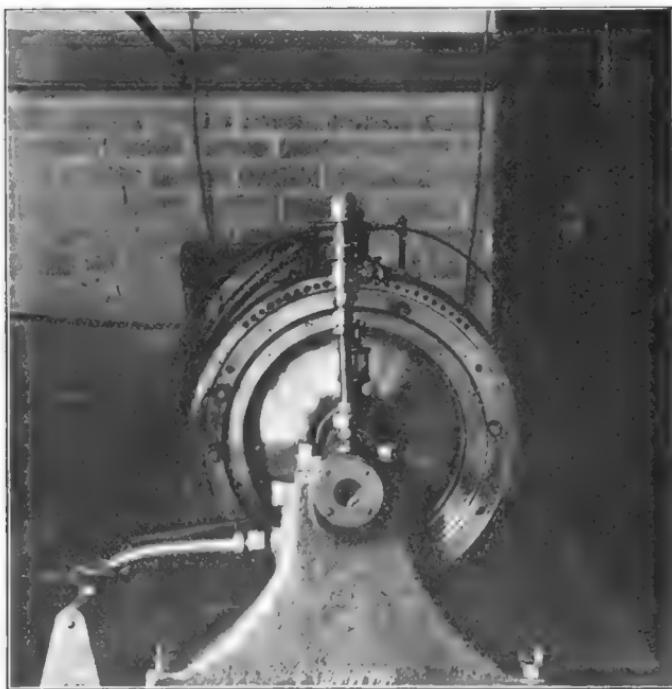
FIG. 89.

supposed to break before anything else goes wrong.
It does not always do this, however. In starting up
the generator to speed there is a critical point which
must be passed very quickly or destruction will result,
for at that point dangerous vibrations are set up due
to mechanical resonance.

Condenser Discharge.

The most common and inexpensive method of pro-
ducing high-frequency currents is the **condenser dis-**

charge method. One way is to charge a condenser from the secondary terminals of a high-voltage transformer (such as previously described), and then discharge the condenser through the primary coil of a high-frequency transformer.

A diagram of the connections between the transformer condenser and high-frequency coil is shown in Fig. 90.
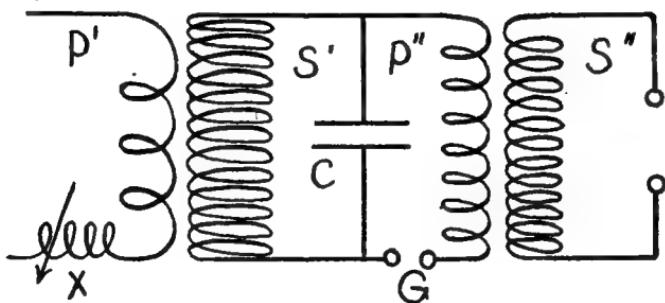
FIG. 90.

The appearance of a spark between the terminals of the 60-cycle, low-frequency transformer is a quiet, yellow flame, Fig. 91, before the condenser is connected to the terminals. The flame is very hot, heating the terminal wires to a white heat; and even melting them in a short time. When the condenser is connected to the transformer terminals, the spark changes from a quiet yellow flame to a roaring spark of dazzling brightness, Fig. 92. This is due to the condenser discharge through the gap G, Fig. 90.

When a condenser is discharged through a very high resistance, the positive charge gradually seeps into the negative plate until both charges are neutralized, Fig. 93. When the discharge takes place through a low resistance a new phenomenon manifests itself —

electric inertia — which may best be explained by analogy.

Suppose that two tall jars a and b, Fig. 94, con-
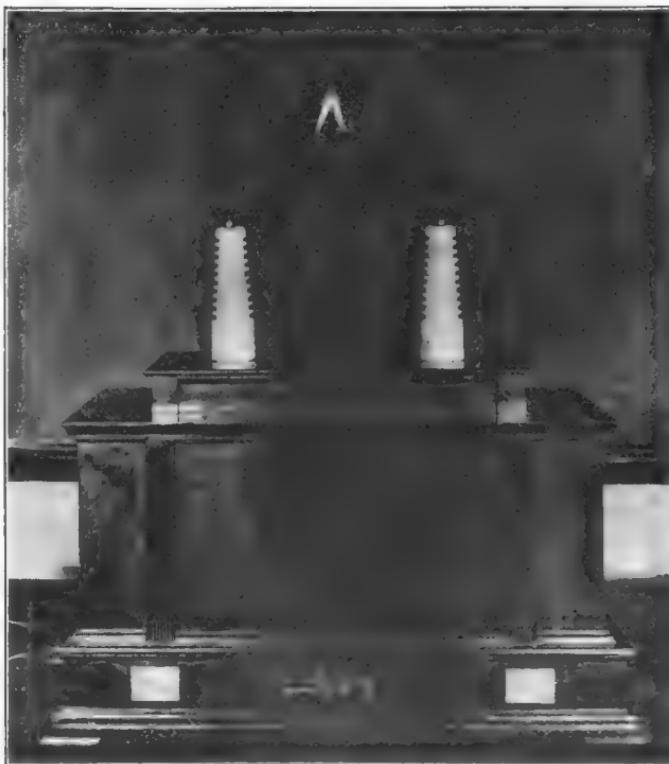
FIG. 91.

nected together at the bottom by a large pipe in which, midway, there is a very thin rubber diaphragm d. Let the jar, a, be filled with water almost to the

top. The diaphragm is strained or stretched next to bursting. Add a little more water to jar a and the diaphragm will burst and permit the water to rush
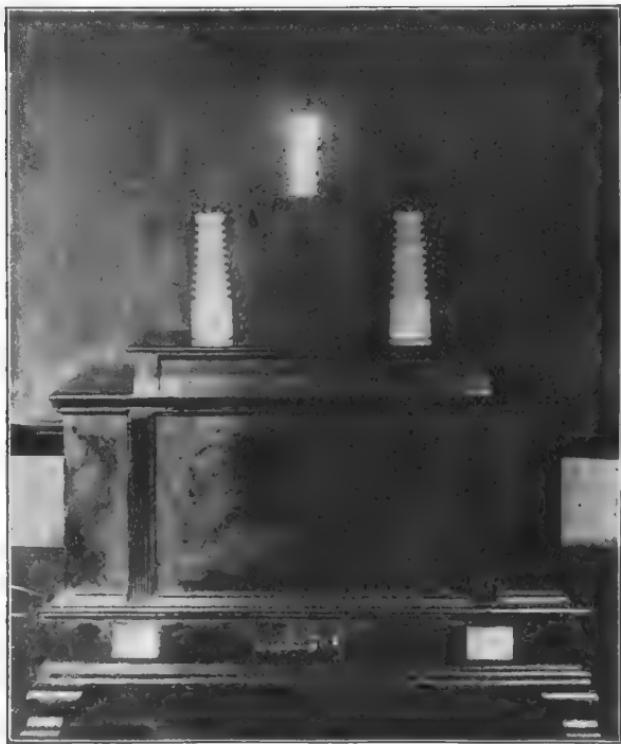

Fig. 92.

into b, but because of inertia the water does not stop until it has reached a point in b almost as high as when it started in a.

It does not stop there, however, as the pipe is wide open, but returns back to a again and reaches a point
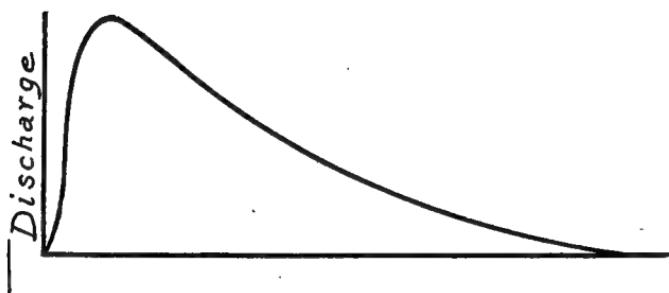
Fig. 93.
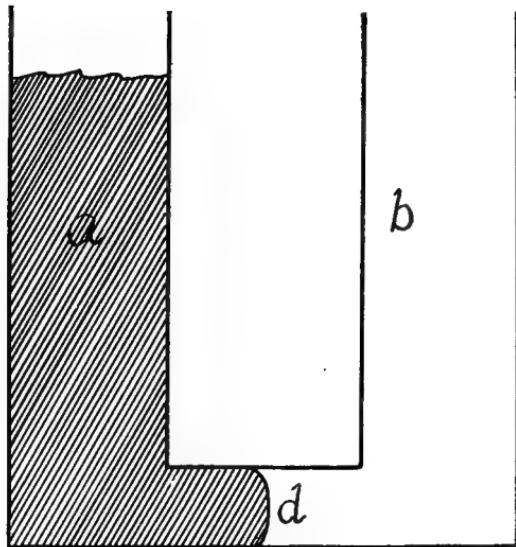
Fig. 94.

a little lower than where it was in b. This rushing back and forth continues until the friction finally causes the water to come to rest at the same level in both jars.

The two jars in the analogue are compared with the two coatings of a condenser, the empty one to the negative, and the full one to the positive. The strain upon the rubber diaphragm is analogous to the electric strain upon the air gap which is punctured when the critical voltage is reached, just as the diaphragm was mechanically punctured when the water pressure got too great.

Increasing the height of the water in the jar a, corresponds to increasing the potential of the charge in a condenser. As increasing or decreasing the strength of the rubber diaphragm determined whether it would burst at a high or low pressure of water, so likewise lengthening or shortening the air gap determines whether the condenser shall be discharged at a high or low potential.

The diaphragm permitted no water to pass until it burst suddenly, after which it offered practically no resistance to the flow; so in like manner the air gap permits no current to pass until a critical puncture voltage is reached, after which practically no resistance is offered to the current until the condenser is discharged.

It is evident from the analogue that the charge does not come to rest at the first discharge; but, because of its apparent inertia, continues to surge back and forth between the condenser coatings. This produces an oscillating current in the discharge circuit, the frequency being exceedingly higher than that of the water oscillations in the jars. A graphical representation of an oscillating condenser discharge is shown in Fig. 95. The high frequency is not determined by the frequency of the charging current.

In the analogue each succeeding oscillation was a little lower than the one that preceded it because of losses due to friction, so likewise the amplitude of each succeeding current wave is less than that of the preceding one because of losses by heat in the resistance of the wire and spark gap, sound and other causes.   Fig. 95.
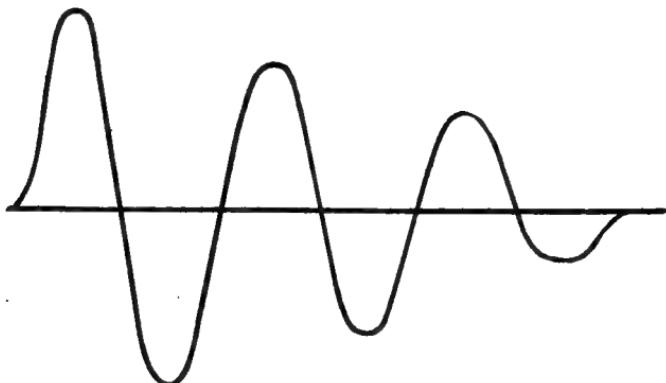

FIG. 95.

When the resistance of the discharge circuit exceeds a certain value the charge will not oscillate. This critical value has been reached when R is greater that $2\sqrt{\dfrac{L}{C}}$.   When the capacity, inductance and resistance are known the frequency can be obtained by the equation

$$f = \frac{1}{2\,\pi}\sqrt{\frac{1}{L\,C} - \frac{R^2}{4\,L^2}}$$

If the resistance is so small that it can be neglected

$$f = \frac{1}{2\,\pi\,V\overline{L\,C}}$$

As most capacities are reckoned in microfarads, and as the inductance can be changed to centimeters the formula becomes simpler, for then

$$f = \frac{5,000,000}{\sqrt{\text{capacity in m. f.} \times \text{inductance in cm.}}} \text{approximately.}$$

The condenser used in the previous experiment
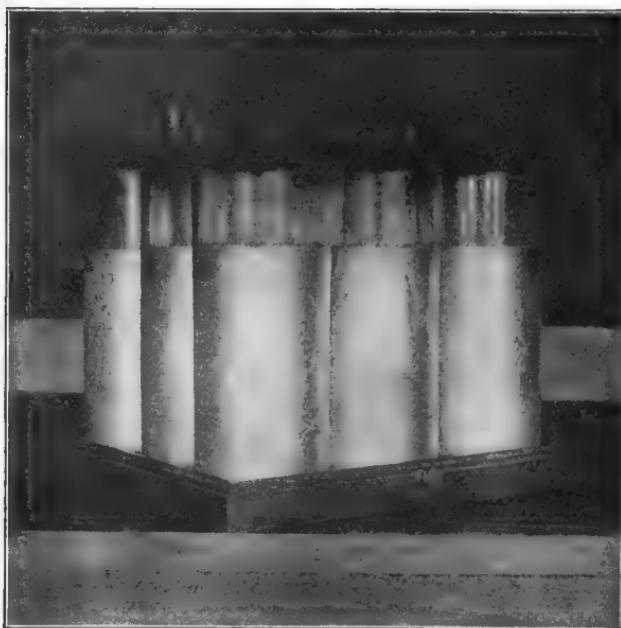
FIG. 96.

consists of nine leyden jars (two quart size) connected in multiple. Fig. 96. Each jar has an approximate capacity of .002 m. f., so all together have a capacity

of .018 m. f.   With an inductance of 1388 cm. the frequency becomes

$$f = \frac{5,000,000}{\sqrt{.018 \times 1388}} = \frac{5,000,000}{\sqrt{25}} = \frac{5,000,000}{5} = 1,000,000 \text{ cycles.}$$

This frequency is not maintained continuously, because a certain time is required by the impressed e. m. f. to reach a sufficiently high potential to break down the high resistance of the air gap.   If the condenser is very large, the e. m. f. may not be able to raise the potential high enough in the first or third quarters of the cycle to jump the gap unless it is shortened.   If the gap is made long, only one train of waves may result, even though the condenser is not too large, Fig. 97, but with a shorter gap several trains
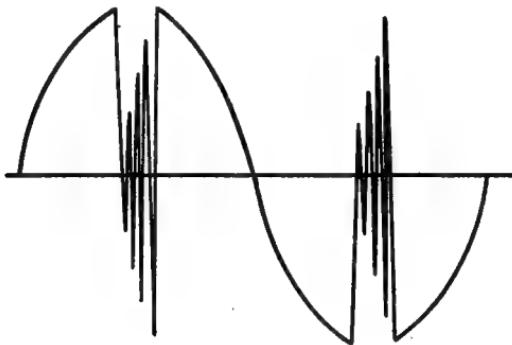
FIG. 97.

of oscillations may take place per half cycle.   When this condition exists the air gap regains its normal high resistance several times per half cycle as soon as the condenser is discharged.

More energy is stored up in a condenser when the spark gap is long, but as it is only charged once or twice per half cycle the total available energy may be

less than when a short gap is used, because then the charging and discharging takes place many times per half cycle, although not as much energy is stored up in each charge.

The length of the spark gap, however, is determined by the required service of the high-frequency current. If long sparks are required from the secondary of a high frequency transformer the spark gap in the condenser circuit should be long, but when a high-effective value of the current is required the gap should be shortened somewhat. When the gap is too short a flaming arc takes the place of the brilliant loud spark and no oscillations are created.

The secondary of the low-frequency transformer is practically short-circuited by the arc so that the difference in potential between the transformer terminals is never high enough to charge the coatings of the condenser. By increasing the inductance of the variable reactance coil $\times$ in Fig. 90 (same as in Fig. 69) the spark will resume its effectiveness even though the gap is short. An efficient spark is loud but uniform. Because of the distressing noise of the spark, the gap is often enclosed in a sound-proof compartment.

Because of the exceedingly high frequency of the oscillatory discharge of a condenser, the inductive effects of these currents is remarkable considering the small quantity of electricity stored in the condenser.

The capacity used in the previous experiment consists of nine leyden jars (two quart size) connected in multiple. Each jar has an approximate capacity of .002 m. f., so all nine together would have about a capacity of .018 m. f.

When charged to a potential of 20,000 volts, the condenser would hold a charge of $Q = 20,000 \times .000000018 = .00036$ coulomb, which can do the work of

$$\frac{.000000018 \times 20,000 \times 20,000}{2} = 3.6 \text{ joules.}$$

With the frequency of 1,000,000 cycles there are 2,000,000 alternations per second, thus the condenser is discharged once during every alternation, or in $\dfrac{1}{2,000,000}$ sec. The average value of the current required to discharge .00036 coulomb in $\dfrac{1}{2,000,000}$ second is 2,000,000 $\times$ .00036 $=$ 720 amperes. Several alternations may be required before the entire energy of 3.6 joules is dissipated.

If eight alternations are required to bring the charge to zero, the 3.6 joules will be dissipated in $\dfrac{8}{2,000,000}$ or .000004 sec. which is at the rate of $\dfrac{3.6}{.000004}$ or 900000 joules per second or 900 kilowatts, so we come to the conclusion that although the energy stored up in the condenser is insignificant the rate of doing work or giving up of this energy becomes enormous when it is given off in a very short space of time.

Although the current in the discharge circuit under consideration is about 720 amperes the heating effect upon the wires is not by far as great as 720 amperes direct current, because, during the part of the low frequency cycle that the condenser is being charged, only a fraction of an ampere flows through the circuit, consequently the wire cools off somewhat during the comparatively long periods of charging in each of the 120 alternations per second.

Although to keep the conductors cool it is not necessary to have as big wires in high-frequency discharge circuits to carry the same number of amperes as with direct current, the R I^2 loss becomes excessive unless very large conductors are used.

This loss is much greater in the same conductor when carrying high-frequency currents than it would

be with direct current because the former does not flow uniformly through the entire cross-section of the conductor but only flows on and near the surface. This phenomenon is called the skin effect. The reason for this is that it takes time for the current to soak into the wire, figuratively speaking, and as this time is not given, the current can not enter the center of all except very small wires; thus the virtual cross-section of a conductor for high-frequency currents is only the area of the edge of a copper tube whose wall thickness is determined by the frequency. With a frequency of a million the current penetrates only about ⅛ millimeter deep in a copper wire and very much less in iron.

The high-frequency resistance of round copper wires straight or nearly straight is $R^1 = R \frac{\pi d}{80} \sqrt{f}$

where R is the natural resistance to a direct current, d is the diameter of the wire, and f is the frequency. Let d equal 1 centimeter, about the diameter of a number 00 wire, and let f equal 1,000,000, then

$$R^1 = R \frac{\pi \times 1}{80} \sqrt{1,000,000} = R \frac{3.1416}{80} \times 1,000 =$$

39.29 R. Thus the high-frequency resistance is 39.29 times the direct-current resistance.

To lower the high-frequency resistance without having unnecessarily heavy conductors, stranded wires should be used because of the increased surface area. The h. f. resistance of very small wires of nonmagnetic material does not differ very much from their natural resistance and the current nearly penetrates to the center of them.

Since the high-frequency current only traverses the skin of a conductor, the inductance with such currents is not the same as with low-frequency currents.

The inductive reactance for high-frequency cur-

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rents is enormous, so enormous in fact that a difference in potential of thousands of volts may exist between two points on a conductor only a few feet in length.
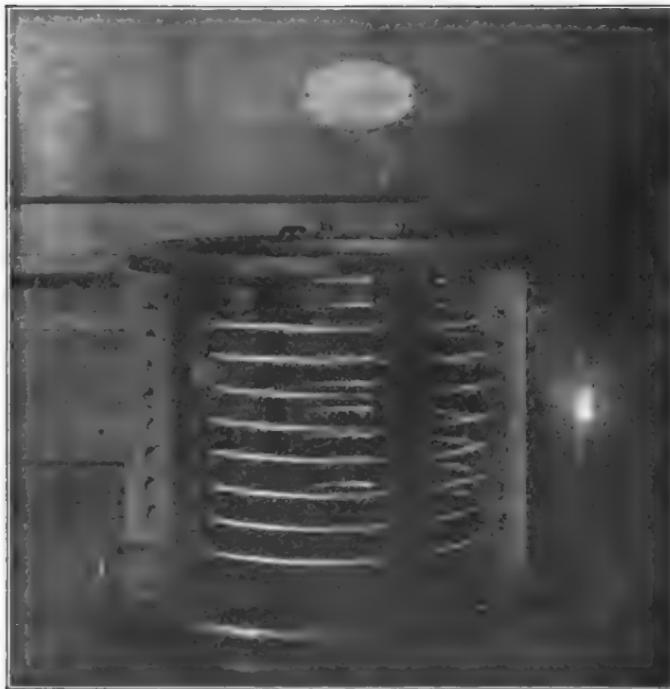
FIG. 98.

Experimental proof of this is shown in Fig. 98, where a discharge of about 15,000 volts occurs between the ends of a helix of about 25 feet of No. 4 aluminum wire.

A diagram of the connections to the helix is shown in Fig. 99.

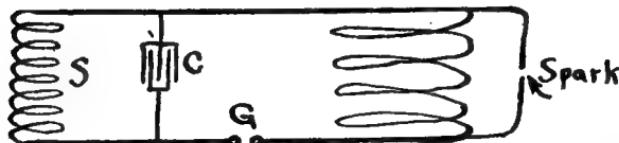Another remarkable example of potential drop along a conductor is shown in Fig. 100, where a 10-volt

FIG. 99.

FIG. 100.

5-watt Mazda lamp is lighted to full candle-power, although apparently short-circuited by 6 inches of No. 00 copper wire.

The natural resistance of a No. 00 copper wire 6 inches long is .0000389 ohm, while the natural resistance of the lamp filament is 20 ohms. These two resistances are really connected in parallel, so that part of the current goes through the copper conductor while the remainder goes through the lamp. If the experiment in Fig. 100 were performed with direct current, with an e. m. f. of 10 volts at the lamp terminals, the part of the current that would go through the lamp would be $\frac{10}{20}$, or $\frac{1}{2}$ ampere, while the part that would go through the 6 inches of No. 00 copper wire would have to be the enormous current of $\frac{10}{.0000389}$, or 257,069 amperes. This current strength, however, would never be reached, as the copper would be melted almost instantly.

It is evident from this comparison that the natural resistance of the copper conductor is insignificant when compared with the resistance due to high frequency inductive reactance. The filament of the lamp also has an enormous resistance, due to inductive reactance; but since the natural resistance of either the copper or the filament does not cut much figure in the impedance, the current flowing through the copper is not many times greater than that through the lamp. That which governs the amount of current most in either circuit is the inductance.

The inductive reactance of the lamp filament is so great that several hundred volts are required to light the 10-volt lamp, so we see that there is a difference of potential of several hundred volts between two points on the No. 00 conductor, although these points are only 6 inches apart.

The filament of the lamp used in the experiment is in the form of the letter V, so is quite well adapted for high-frequency lighting because of its low inductance. A perfectly straight filament has the least possible inductance and is therefore the best adapted of any for high-frequency currents. Where the filament is formed into a helix of two or more turns, or into a spiral like a clock spring, the inductance increases rapidly, and, consequently, the inductive reactance. With a higher reactance a higher e. m. f. is necessary to light the lamp to full brightness. The voltage may have to be so high that sparks will jump across the space between the leading-in wires in the neck of the globe rather than go through the few inches of coiled filament.

Another advantage of a single loop, or a straight filament, over one having two or more turns, is the absence of vibration. With the common kind of 110-volt filament of two turns the vibration due to mutual reactions between the two turns (and other causes) is sometimes so violent as to tear even a thirty-two candle-power filament to pieces.

As commercial incandescent lamps are rather highly exhausted of air, that which remains is in a very rarefied state. When a lamp is lighted with high-frequency currents this rarefied air sometimes glows with a faint bluish light. In new globes the light is not always perceptible, but globes that have been used with high-frequency currents several times give off considerable of the bluish light at the moment the current is turned on. Sometimes the blue glow is visible even while the lamp is burning at full candle-power. This effect shortens the life of the lamp very much.

The vacuum of commercial lamps is so very high that very little current can pass through it. With a much lower vacuum, however, it is possible to send more current through the rarefied air than through the

carbon filament, so that the filament remains dark while the vacuous space around it glows with a beautiful violet-colored light. Thus we see that air at a certain low vacuum becomes a good conductor.

The long glass tube shown in the lower portion of Fig. 100 is a low-vacuum tube, such as is used by doctors when giving high-frequency treatments. The color of the light given off by this tube is a rich violet. Metal terminals sealed within a tube of this kind are unnecessary, although some manufacturers do seal a terminal within one end only. The tube in the photograph, however, has none, it being lighted entirely by induction through the glass of the tube. A piece of tin-foil wrapped around one end and a brass ferrule on the other end serve as terminals.

The tin-foil and ferrule act as two coatings of a condenser, with a double glass dielectric and conductor interposed between them, as shown in Fig. 101. The interposed conductor is the rarefied air.
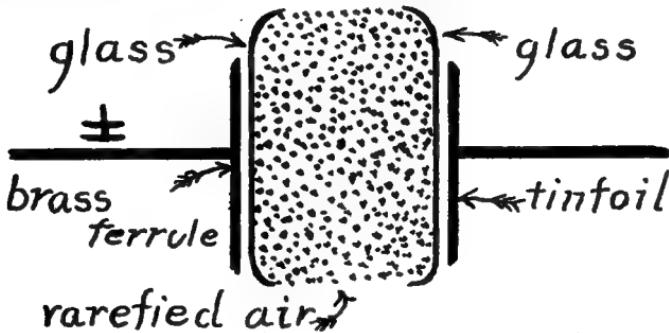
FIG. 101.

The circle of No. 00 wire in Fig. 100 is connected in series with a condenser and a spark gap, as shown in Fig. 102, where S is the secondary of a high-voltage

transformer, C is a condenser of Leyden jars, and G is the spark gap.

Another method is shown in Fig. 103, where two condensers are used in series.

The remarkable inductive effects of high-frequency currents is well illustrated in Fig. 104, where a 10-volt, 5-watt Mazda lamp is lighted to full brightness through
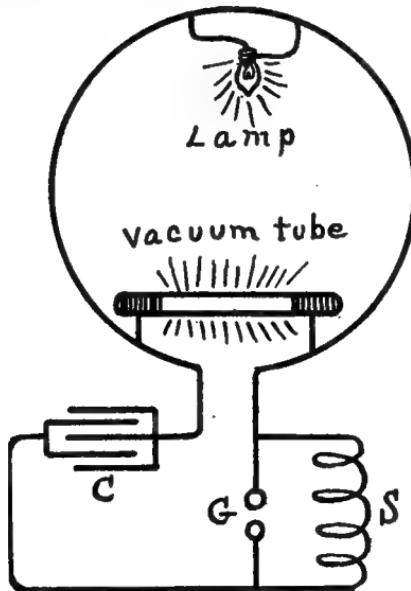
FIG. 102.

a cement wall, a wireless electric light, if you please. The receiving end consists of one turn of No. 10 aluminum wire, wound about a circle of nails driven into the plaster. The circle is 4 feet in diameter. The 10-volt globe was inserted between the two ends of the turn which was connected to the two terminals

of the lamp filament. In fact, the receiving end is no more or less than the secondary of a very loose-coupled transformer.

The transmitter, or more properly the primary of this transformer, consists of two turns of ¾ copper
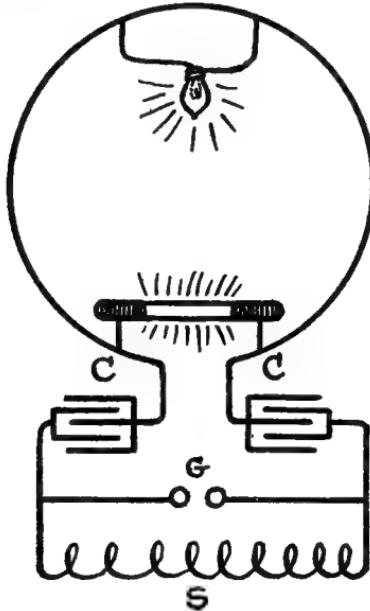
FIG. 103.

ribbon wound on a square frame 7 feet to each side. Large wire would have been just as good for the conductor as the ribbon, but would have been heavier. The two turns on the frame are connected in series with a condenser and a spark gap, as shown in Fig. 105. The secondary of the low-frequency step-up transformer is represented by S.
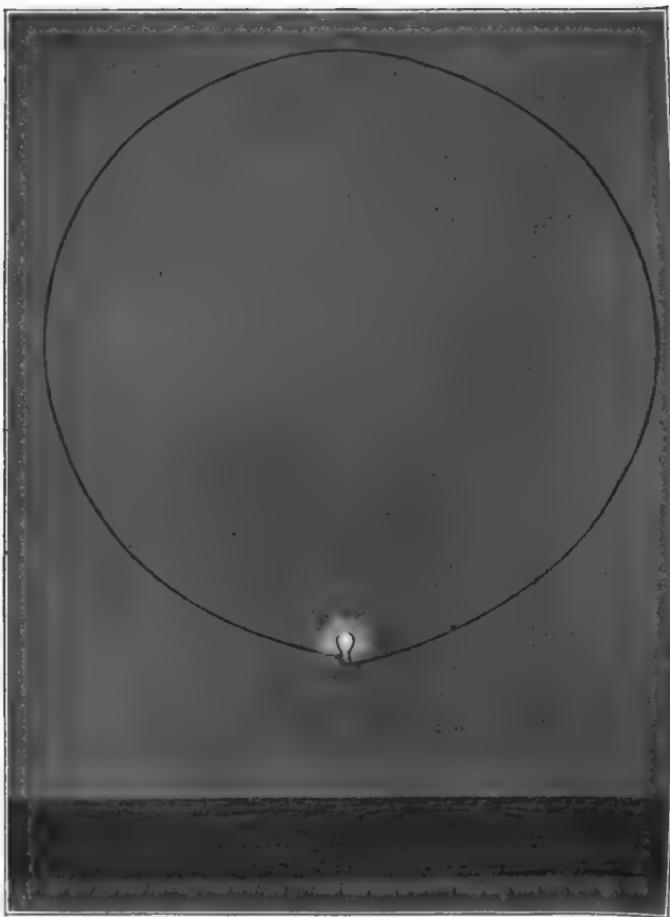
Fig. 104.

This primary coil was placed in a room on the other side of the cement wall shown in the photograph. This coil was set in the same plane as the secondary coil on the wall and about 4 to 6 feet back from it.

For demonstration purposes it is more convenient to place the one-turn secondary on a form about 4 feet square. This form can be made in the shape of a cross, the same as in the primary, Fig. 105. With a
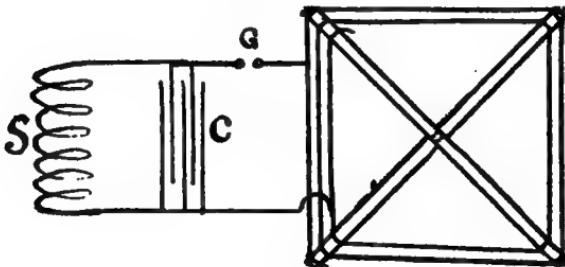
FIG. 105.

portable secondary of this kind the light can be carried about the room, the globe burning brilliantly as long as the secondary is held in the same plane as and not too far from the primary, and most brilliantly when held close to and directly opposite the primary. Care should be taken not to approach too closely, or the lamp will be burned out. With 110 volts, 60 cycles, impressed on one-half of the primary of the step-up transformer, the globe may be lighted nicely 5 or 6 feet away from the wireless primary or transmitter, providing the secondary frame is held directly opposite it. A variable-reactance coil such as shown in Fig. 74 is connected in series with the primary of the step-up transformer, to limit the primary current so as to prevent an inactive arc in the spark gap of the oscillatory circuit.

With 220 volts impressed on half the primary of the step-up transformer, and with a larger wireless receiving secondary of one turn of 7 feet square, the globe may be lighted at a distance of 10 feet or more
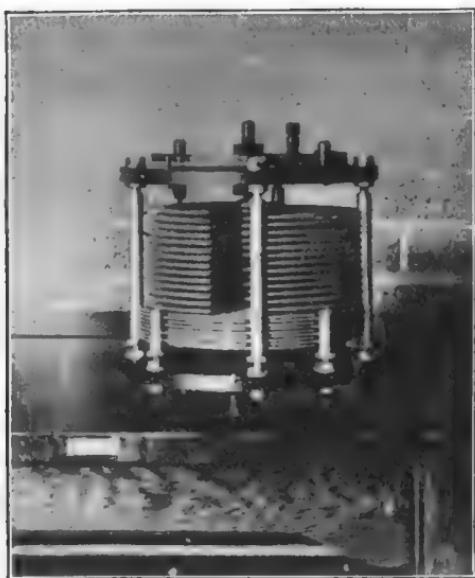
FIG. 106.

from the transmitting primary. The low-frequency current must be controlled by a suitable reactance.

The wireless light may be extinguished at will by turning the secondary at right angles to the plane of the primary coil, for in this position no lines of force thread through the secondary turn. With a greater number of turns on the secondary a higher voltage lamp may be lighted.

Because of the high self-induction in the receiving coil only a fraction of the generated e. m. f. is used to light the lamp. By inserting a variable condenser, Fig. 106, and a variable inductance in series, the lamp and secondary turn, the condenser and inductance can be so adjusted that a condition of resonance will exist and the lamp be made to burn brighter or at a greater distance from the primary.

The variable condenser consists of a set of semi-circular aluminum or copper plates, spaced ¼ inch apart and mounted on a vertical spindle, and a set of stationary semi-circular plates also spaced ¼ inch apart. The plates on the spindle can be rotated so that they pass between the stationary plates without touching them. When the plates on the spindle are turned entirely out from between the stationary plates the capacity is very small, but when the rotating plates are entirely between the stationary plates the capacity is increased to its maximum value.

The variable inductance can be made by winding a single layer of No. 10 bare copper or aluminum wire on a wooden rolling-pin 2 or 3 inches in diameter. The turns should be spaced apart so that adjacent convolutions do not touch each other. With a sliding contact upon this inductance the inductance may be varied in quite small steps.

Although there are very few lines of force mutually interlinking the primary and secondary of the wireless light transformer, yet the e. m. f. generated in the secondary is considerable because of the exceeding high rate of expansion and collapse of these lines.

When a condenser is simply connected across the terminals of a step-up transformer, and not allowed to discharge, there may be very little brush discharge at the edges of the tin-foil. If, however, the condenser is allowed to discharge through a coil of low inductance, the brush discharge becomes very great.

This phenomenon can be used in an artistic way, as shown in Fig. 107, where the name, Nikola Tesla, appears in a darkened room, as if written in violet

FIG. 107,

fire. One of the prettiest applications of the brush discharge is shown in Fig. 108, where a large butterfly appears almost lifelike, it being possible to bring out the many beautiful details of its anatomy in a violet-colored spray. To obtain the best effect the experiment should be done in perfect darkness, as then only the spray becomes prominent.

The writing or design is first cut out of tin-foil pasted on a large sheet of glass. A margin of several inches should be left around the design. On the opposite side of the glass should be pasted a sheet of tin-foil, which should extend 2 or 3 inches farther out than any part of the design on the reverse side. When this has been done, the two coatings should be connected in parallel with the condenser, as shown in Fig. 109.

Many very interesting experiments can be performed with a high-frequency coil which will give a moderately high voltage and a very heavy current of, say, 1 ampere effective value. Such a coil is shown in Fig. 110, where a 110-volt, 80-watt carbon-filament

FIG. 108.

lamp is lighted to full candle-power with only one con-
nection to the coil.   Besides, the entire current of
almost an ampere passes into the body through the
mouth.   The lamp is supported by a heavy aluminum
wire held firmly in the mouth.   With the coil running
at its full capacity, the 80-watt or even a thirty-two
candle-power lamp could have been burned out by the
heavy current.   A thirty-two candle-power lamp would
have been used in the experiment, Fig. 110, only one
was not obtainable at the time.

Although the effective value of the current passing
through the lamp and mouth is about 1 ampere, the
maximum current is enormously higher.   When the
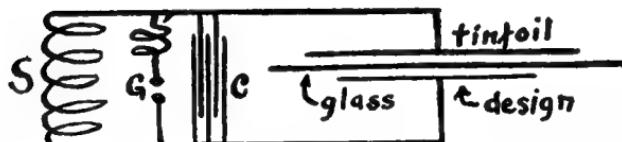
FIG. 109.

FIG. 110.

condenser-discharge gap is adjusted to give the largest effective current in the secondary circuit of the high-frequency transformer the condenser may be charged and discharged several times during one alternation of low-frequency charging e. m. f.

As stated before, each condenser discharge may consist of a group of oscillations of decreasing amplitude. The time required for each group of oscillations is very small compared with the time required to charge the condenser. If the frequency of the discharge is 1,000,000 cycles, then each alternation requires $\frac{1}{2,000,000}$ second, and if there are four oscillations in the group the time required is $\frac{4}{2,000,000}$, or $\frac{1}{500,000}$ second. Assuming that the spark gap is so short that the condenser can be charged 25 times per low-frequency alternation, then there will be 25 $\times$ 4, or 100 high-frequency oscillations per low-frequency alternation, or 100 oscillations in $\frac{1}{120}$ second, since the charging frequency is 60 cycles. In one second there would be 120 $\times$ 100, or 12,000 oscillations.

The part of the second occupied by the oscillations would then be 12,000 $\times \frac{1}{2,000,000}$, or .006 second, while the time required in charging would be 1 — .006 = .994 second. We see from this that the current is only active for .006 part of the time and must be $\frac{1}{.006}$ or 166 times as large to have the same effective value as a steady high-frequency current; in other words, when the effective current through the lamp and mouth is 1 ampere, the average value of the current

given off in each group of oscillations is 166 amperes. Since the amplitude of each succeeding high-frequency alternation of a group (Fig. 95) is less than the one which preceded it, the first oscillations of the group are much higher than the average amplitude, consequently the maximum value of the current which flows through the lamp and mouth is much more than 166 amperes. No shock is felt from this heavy current; in fact, no sensation of any kind except heat, in case the contact in the mouth is not large enough. Although it is not known exactly why a person is not harmed by moderately heavy high-frequency currents, nor why a person does not feel them, yet there are two very reasonable theories advanced to account for these phenomena.

One theory is that the nerves, fast as they are, are too slow to respond to current of a frequency of 10,000 cycles and upward. If the current, in passing through the body in one direction affects the nerves, the current as it reverses neutralizes the effect of the first half cycle before the nerve has time to respond.

Another theory is that since the currents of very high frequency only penetrate very slightly the surface of the body of a conductor, the current never penetrates deep enough to affect the nerves.

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CHAPTER XIV.

Although both theories may be faulty, yet it is true that currents much heavier than required for electrocution purposes can be passed through the body without even discomfort.

The question might be asked how it is possible to light an electric lamp with only one wire connected

FIG. 111.
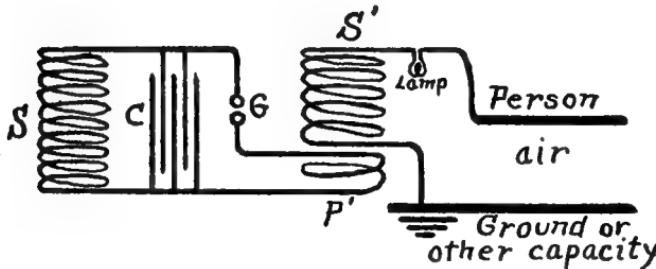
to the coil. The answer is that the body of the person holding the lamp terminal acts as one plate of a condenser; in other words, a capacity.

One terminal of the high-frequency coil is grounded, say, to a water pipe, thus the earth becomes the other plate of a condenser with widely separated conductors and an air dielectric. A diagram of connections is shown in Fig. 111.

.When a good ground connection can not be obtained, the lower end of the secondary coil S^1 can be connected to the primary coil P^1, which in turn is con-

nected to the Leyden jar condenser. This gives some capacity, but probably not enough sometimes, so it may be necessary to connect the ground wire to an

FIG. 112.

extra capacity made from a large sheet of metal (tin will do) laid on the floor some distance away or suspended in the air.

The primary helix of the coil used in the preceding and some of the following experiments consists of

eight turns of No. 4 aluminum wire, the adjacent turns being spaced 1 inch apart. The helix, 12 inches in diameter, is held in shape by being threaded through

FIG. 113.

a number of holes bored through six vertical strips of wood, as seen in Fig. 110. The strips of wood, 10 by 2 by 1 inches, are arranged in a circle and their

lower ends screwed fast to a nicely finished base, the top ends being surmounted by a large fiber ring.

The secondary coil is wound in a single layer upon a pressboard paper cylinder 7½ inches in diameter and 9 inches long. Seventy turns of No. 18 bell wire or rubber-covered fixture wire are wound in this layer.

FIG. 114.

The fixture wire is to be preferred because of its better insulation.

The secondary is concentric with and set within the primary coil, so that the lowest turn of the secondary is about 2 inches lower than the top turn of the primary. Using only the four top turns of the primary and only five of the Leyden jars gives the best results in the experiment Fig. 110.

Only half of the primary of the low-frequency step-up transformer is used in this and all the rest of the experiments. A reactance coil is absolutely neces-

FIG. 115.

sary for the successful working of high-frequency apparatus, unless the primary and secondary of the step-up transformer are very loose-coupled, as in Fig. 74. The object of having a reactance in the primary

circuit is to prevent a flaming discharge in the spark gap without lowering the secondary potential very much. A reactance coil is better than a noninductive rheostat for this reason and because the reactance is more economical with the current.

The yoke of the reactance (Fig. 69) was removed and 220 turns used in series with the primary of the step-up transformer in the preceding and most of the following experiments. The size of the low-voltage supply transformer determines to some extent how much reactance is necessary. If the transformer is large, more reactance is needed than when the supply transformer is no larger than the step-up transformer. The larger the condenser shunted across the secondary the less is the reactance required in the primary circuit.

In the experiment in Fig. 110 a large sheet or body of metal suspended in the air or set upon some insulating support can be substituted for a person's body, with just as good or better results if the metallic body can be adjusted in size. Since the inductive reactance of the secondary of the high-frequency coil is exceedingly high, less current will flow through it on short circuit than on open circuit with suitable capacities attached to its terminals. An incandescent lamp connected directly across the secondary terminals might burn only a dull red, but when a condenser (of two 10-inch square metal plates, separated by a sheet of window glass) is placed in series with the lamp and secondary, the lamp burns very brightly because the capacity neutralizes the inductance, which causes a more nearly resonant condition to exist.

It is more difficult to light a lamp taking only $\frac{1}{2}$ an ampere between two persons, as shown in Fig. 112, than an 80-watt lamp lighted as in Fig. 110, as only a fraction of the current passes into the second person. The first person grasps the terminal of the secondary

firmly in one hand, while the globe to be lighted is held out to the second person with the other hand.

FIG. 116.

As more than 1 ampere (effective value) of current flows through the hand holding the secondary terminal, it is necessary to have a large contact surface,

such as a metal tube an inch in diameter and 5 inches long. Even the palms of the hand may be heated severely if the current is held on for a minute or so at a time.

Vacuum tubes, substituted for the lamp in Fig. 112, can be lighted very brightly, as shown in Fig. 113.

FIG. 117.

Not only is the current very large which flows into the person who grasps the terminal, but it is also of very high potential, yet the heavy high-voltage discharge can be taken into the body by a wire held in the mouth (Fig. 114). The voltage of the discharge in the photograph is about 50,000 volts. Were this discharge to be taken directly upon the skin, great pain would be felt and the skin probably blistered or seared. There is, however, no bad after-effects from such a burn as there is with one caused by X rays.

The difference in potential of two different places on a person's body when connected to the terminal of the coil is well illustrated in Figs. 115, 116, 117. In the first experiment a lamp which requires ½ ampere is lighted between the two hands as though the current were generated in the body itself. The lamp
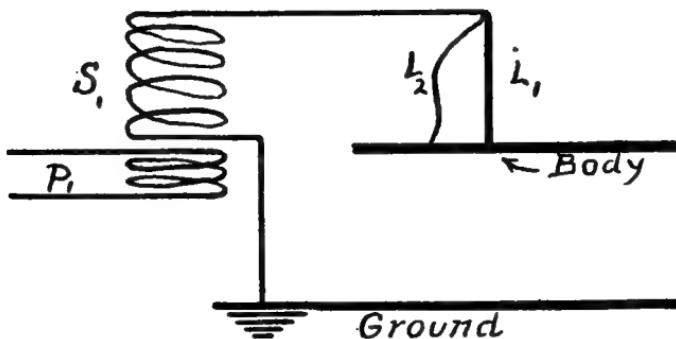
FIG. 118.

burns much brighter, however, if held between the mouth and one hand (Fig. 116). A bright spark may be drawn between the tips of the fingers, as shown in Fig. 117.

In these experiments the current is conducted to

the body proper, in a divided circuit of two paths. The main path is through the hand and arm in direct contact with the coil terminal, while the other path is from the right hand through the lamp or gap and left arm in Figs. 115 and 117, and from the right hand through the lamp and mouth in Fig. 116. The path through the lamp and mouth has less inductance than that of the lamp and arm. This explains why the lamp burns brighter in Fig. 116 than in Fig. 115. The plan of the divided circuits used in charging the body is shown in Fig. 118, where L_1 represents the right arm or main path, while L_2 represents the lamp circuits through the mouth or left hand.

A striking manifestation of the high potential to which a person may be charged is shown in Fig. 119, where sparks 10 inches long extend from the tip of one index finger. The sparks look like forked tongues of violet flames, which squirm around like serpents dancing on the tips of their tails.

If a vacuum tube were held in the left hand (Fig. 119) the tube would shine brilliantly, as shown in Fig. 120. The end of the tube is covered with streamers 5 or 6 inches long, which emanate from the surface of the glass.

Instead of a regular vacuum tube an ordinary incandescent lamp globe can be used. A globe whose filament is burned out, but which has a good vacuum, will do just as well as a new globe. The light of the globe is at first an intense blue, but after being run a few moments the bulb may puncture at the tip and the vacuum lowered until it finally reaches atmospheric pressure. While the vacuum is passing through these changes the color of the light changes from light blue to a rich reddish violet.

After the vacuum has been completely destroyed the current no longer leaves at the surface of the glass,

Fig. 119.

FIG. 120.

but at the ends of the filament, which is heated to incandescence and rapidly disintegrated.

If a vacuum tube is held in the mouth it will glow quite brightly, but not as bright as when held in the

Fig. 121.

extended left hand. If both hands are held down and the head thrown back so that the nose points upward, sparks 1½ inches long stream from the tip of the nose. When the head is held in its normal position sparks

may be seen leaving the tips of the ears. Even the top of the head is covered with a bluish spray. The room should be very dark to perform these experiments to the best advantage.

If a second person should approach too close to the one who is so highly charged, a spark 6 to 8 inches long would jump to him (Fig. 121). This shows that the potential of the charged person is about 100,000 volts. A vacuum tube held by the second person at a distance of several feet from the charged person will light up entirely by induction.

The connections from the secondary terminal of the high-frequency coil to the charged person (in experiments Figs. 119, 120 and 121) were made to his ankles. To insure a good contact with the skin, a metal band an inch wide was wrapped around each ankle. The bands were connected by a wire to the coil. The last three experiments were made with a different secondary coil than those preceding. A higher voltage was needed, so 125 turns of No. 18 fixture wire were used instead of the 70 turns of bell wire as described previously. Seven top turns of the primary were used in series with five of the Leyden jars to produce the best results.

The lower end of the secondary coil was placed about 2 inches lower than the top turn of the primary, so was rather loose-coupled. To obtain the longest streamers from the fingertips, and, in fact, all the last three experiments, the step-up transformer was run by 220 volts properly controlled by the reactance coil.

Since the person in Fig. 119 was so highly charged, it was necessary for him to stand on a chair, as 6 or 7 inch sparks streamed out through his shoes when standing on a box 8 or 10 inches high above the floor. The sparking through the shoes is very disagreeable.

By using only the two top turns of the primary coil, and having considerable inductance in the primary cir-

FIG. 122.

cuit of the step-up transformer, the whole top of the high-frequency secondary is covered with a beautiful spray so fine and dense that it looks like the fine spray of a fountain, only colored a bluish violet.

There seems little tendency to make a real crackling spark to any conductor brought near unless very close. The spray is so fine in quality that even a person's face may be held so close that the spray covers it (Fig. 122). If held too close the sensation is unpleasant, like the pricking of a thousand needles, but when held just close enough to feel the spray the sensation is rather agreeable. When the inductance in series with the primary of the step-up transformer is lowered, the spray changes in form to long, flaming streamers. This indicates that the primary turns and the condenser are in resonance. When the secondary terminal is touched by a person, in other words, when a capacity is added to the secondary, the primary is thrown out of resonance with the Leyden jars. This state of dissonance can be remedied by using more turns in the primary, say four or five, because the inductance of each turn is less than before because of the reaction of the secondary current upon the primary. The mutual inductive reactance of the secondary current upon the primary inductance acts like a capacity reactance; thus for a coil to work properly when a person holds the terminal, the primary should have a greater inductance on open circuit than is required to obtain resonance on open circuit.

An example of this effect was noticed in a marked manner when a coil, which would give only about a 2-inch spark to a person reaching the hand to the secondary terminal, would produce sparks 5 or 6 inches long to another person when the first person touched the secondary terminal. The primary may be so adjusted that an apparently inactive secondary will produce a great number of sparks between the turns of the secondary and between the primary and secondary, which shows that the capacity in the secondary brought about a resonant condition between the primary and condenser. Likewise a capacity,
15

FIG. 123.
THREE-FOOT SPARK.

FIG. 124.
FOUR-FOOT SPARK.

FIG. 125.
FIVE-FOOT SPARK.

added to the secondary of coil whose primary is in resonance with the condenser in series with it, will throw the primary out of resonance.

High-potential Discharges.

The most spectacular and fascinating experiments of high-frequency currents is the production of very high-potential discharges, as shown in Figs. 123, 124,

FIG. 126.

LONG DISCHARGES FROM ONE TERMINAL.

125, 126. The photograph in Fig. 123 is that of a very large spark, 3 feet long. In Fig. 124 the spark is 4 feet long, but not quite as heavy as the one in Fig. 123.

A photograph of the high-frequency transformer which produced these sparks is shown in Fig. 127. The ball terminals are situated slightly more than 5 feet apart, yet the coil was able to produce a heavy spark spanning the entire 60 inches, besides arching upward

almost 2 feet higher, so that the real length of the
longest spark was about 75 inches, the e. m. f. being
at least 1,000,000 volts (Fig. 125). The noise was
terrific.

One of the strangest displays of high-potential
high-frequency currents is shown in the remarkable

FIG. 127.

ramifications of a discharge issuing from a single ball
terminal (Fig. 126). When the opposite terminal is
connected to the central portion of the secondary to
prevent it from sparking, the single terminal dis-
charges a torrent of branching sparks about 3 feet
long.

A person standing on an insulated platform or

chair can receive these long discharges of about 500,000 volts into the body. A metal rod should be held in the hand presented to the discharge, so that the spark will not strike directly upon the skin.

FIG. 128.

The photograph in Fig. 128 shows a very uniform brush discharge between a copper wire circle and a point at its center. The circle is connected to one terminal of the coil, while the point at the center of the circle is connected to the other terminal. The circle is about 20 inches in diameter. When the coil is run a dense violet spray fills the space between the point and the circle. Great quantities of ozone are produced by the brush discharge. The inductance in the primary circuit of the step-up transformer should be quite large, to prevent heavy sparks from jumping from the point to the circle in Fig. 128.

A piece of wire bent in the form of the letter S, as shown in Fig. 129, will revolve with great rapidity

S shaped wire

S wire mounted on top of a needle fastened to terminal.
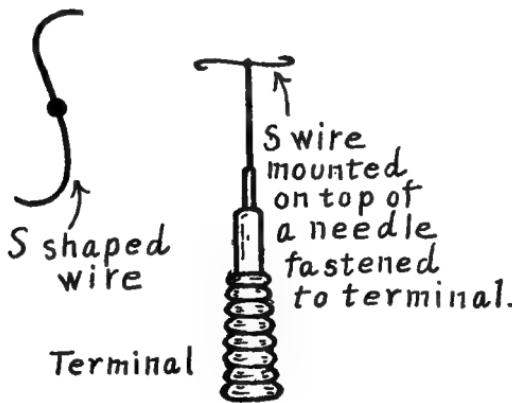
Terminal

FIG. 129.

when balanced upon one of the coil terminals. High-potential discharges have tendency to leave at the points of conductors rather than flat or round surfaces, therefore the sparks from the terminals leave at the ends of the S-shaped wire. The air particles surround-

ing the ends become charged at any instant to the same sign as the wire end at the same instant. Thus the air particles, being of the same sign as the wire end, are repelled away from the wire, and in like manner the wire end is repelled by the air particles surrounding it. This causes the rotation of the S-shaped wire.

Two remarkable things about a high-frequency transformer, capable of producing the enormous e. m. fs. just described, is the ridiculously small amount of wire in the secondary and the absence of an iron magnetic medium.

The primary and secondary of this transformer are wound in concentric single-layer coils, as shown in Fig. 130. The primary is wound upon two fiber pails, each pail being 10 inches in diameter at the top, 8 inches in diameter at the bottom, and 11 inches high. Fifteen turns of No. 4 solid copper wire are wound on each pail. The two coils thus formed are connected in series so as to form a helix, the turns of which all go in the same direction. Wires Nos. 1 and 2 (Fig. 130) are connected together to accomplish this. The two remaining ends of the primary are connected to binding-posts P_1 and P_2.

The secondary coil is wound in two sections, each section being wound in a single layer upon a glass jar 26 inches long and $5\frac{1}{2}$ inches in diameter. Holes were bored through the bottoms of the jars and the fiber pails, after which the jars were bolted to the pails, as shown in Fig. 130.

No. 24 triple silk-covered wire was used in winding the secondary. The turns are wound close together in a single layer 20 inches long on each jar. The two sections are connected in series, and from their junction 3 they are connected to the primary at 1 (Fig. 130. The direction of secondary windings (Fig. 130)
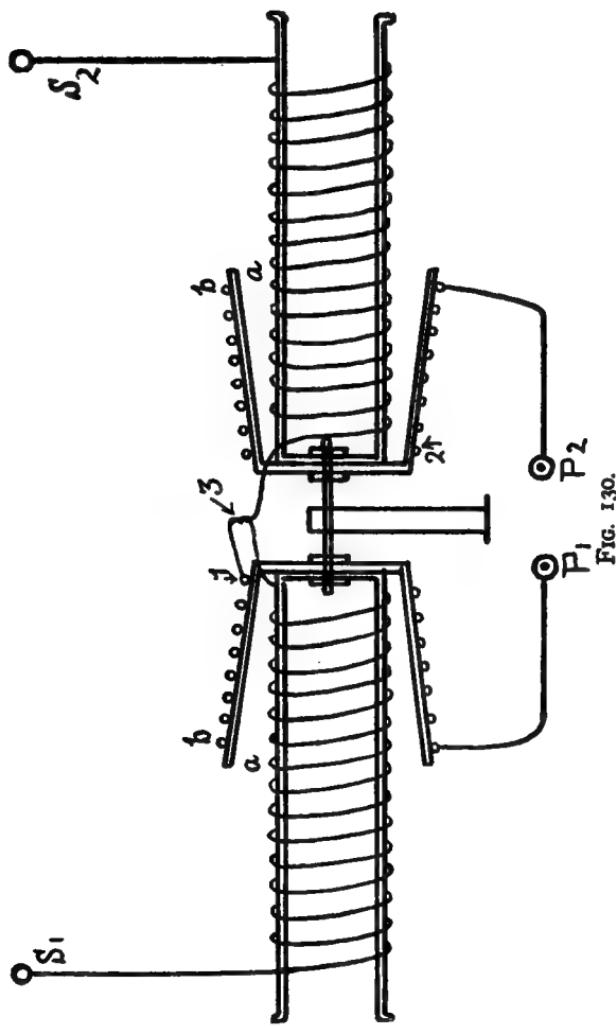
FIG. 130.

should be observed, so that the two sections will work in harmony.

When completed the coils were placed in a case 66½ inches long, 14 inches high and 12½ inches wide, inside measurements.

The bolt which connects the two jars together rests in a notch cut in a board screwed to the bottom of the case, while the ends of the jars are suspended in loops of twine hung from crosspieces of dry birch, which are screwed to the upper edges of the case inside.

The entire case was filled with about 350 pounds of melted petrolatum as soon as the coils were set in position and the terminals brought out at their proper places. The petrolatum must be hot enough to run freely, but not so hot as to get scorched.

Hard insulations such as wax, rosin, etc., are not as well adapted for high-frequency currents as oils or soft insulations. The insulating strength of hard insulations gets less when the frequency of the current is raised, while the insulating qualities of oils and some soft insulations are remarkably higher with high frequencies than with low. Prof. Elihu Thomson states that 1 inch of oil, which could be punctured readily by a low-frequency spark only a few inches long, successfully insulated a 50-inch high-frequency spark.

One handicap in using oil is the difficulty in retaining it in a wooden vessel or case, and another bad feature is that the oil slops around a great deal when the case is moved about.

So that all the air contained in the glass jars might escape when the petrolatum or oil is poured in, the tops of the jars should be set slightly higher than their bottoms.

The ends of the secondary are carried up through the cover of the case through specially made terminals which stand 14 inches high above the top of the case.

These terminals were made hollow and afterward filled with melted petrolatum. To prevent surface leakage the outside of the terminals are deeply corrugated.

A metal rod, which runs centrally through each terminal, is surmounted by a brass ball 1½ inches in diameter.

To keep the terminals well away from the floor, the case is set upon a cabinet about 6 feet long and a little over 3 feet high. The step-up transformer, the condenser, spark gap and reactance coil may all be placed in the cabinet if it is desired to make the outfit more compact. The weight of the entire outfit is about 800 pounds.

The reason that the primary coil is smaller at the middle than at the ends is that closer coupling with the secondary can be obtained in this way without great danger of puncture of the insulation between a and b, Fig. 130.

The potential of a properly designed secondary coil increases gradually from zero at the junction of the two sections to maximum at the terminals.

Since this is so, the potential difference between the primary and secondary also increases gradually from their junction at the middle out to their ends. Since the thickness of insulation is less between a and b than between a and the terminal, the greatest danger of puncture is between a and b.

Since the potential difference becomes less as the middle portions of the primary and secondary are approached, the primary turns can be made smaller, and consequently closer to the secondary without much danger of puncture.

When the wire in the secondary coils is too long the greatest difference of potential may not exist at the ends of the secondary, but at some two interme-

diate places.   The same result is obtained when too small a condenser is used in the primary circuit.

In other words, several potential nodes (zero values) and antinodes (maximum values) may exist on the same coil, providing the wire is long enough.

As the coil is an inductive circuit, the current lags behind the e. m. f., so that where there is a potential node there is a current antinode, and where there is a current node there is a potential antinode.   Such phenomena are called stationary electric waves.

Stationary electric waves on wires may best be
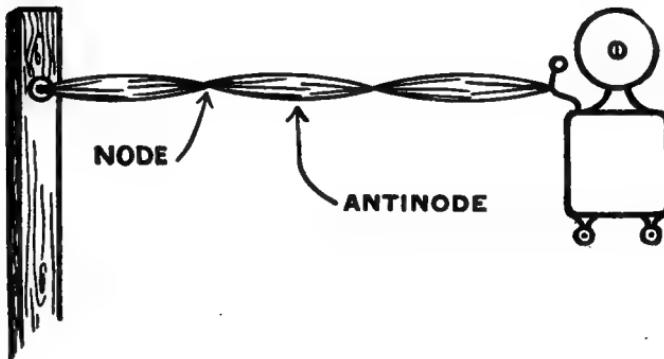
FIG. 131.

explained by analogy.   Every one is familiar with the wave motion produced in a rope which is fastened at one end and periodically moved up and down, or jerked at the other end.   The wave moves away from the one who manipulates the rope to the fastened end, and then returns to the person again because the wave is reflected from the stationary end.   With a proper choice of the number of jerks per second the operator can cause the advancing waves to be interfered by the

reflected waves so that there will be several stationary positions along the rope.

This phenomenon can be illustrated very nicely with a piece of string tied to the hammer of an electric bell at one end and to anything stationary at the other (Fig. 131).

The length of waves produced depends upon the frequency of bell hammer and the velocity of wave movement over the string. The number of stationary waves that can be created on the string is determined by its length and the length of each wave.

Now the velocity of electric waves is not the same when the conductor is coiled in a spiral as when straight or nearly so. When the conductor is straight or almost so, the velocity of wave propagation is the same as the velocity of light, but when the conductor is coiled the velocity is decreased, the factor governing the decrease being $\dfrac{1}{\sqrt{CL}}$ where C is the capacity of the coil per centimeter in length and L the inductance per centimeter where the velocity is measured in centimeters per second.

The wave length of an oscillating e. m. f. can be calculated by the formula $\lambda = \dfrac{1}{f\sqrt{CL}}$, where λ, the Greek letter lambda, represents the wave length in centimeters.

The positions of the potential nodes and antinodes on a long spiral can be visibly demonstrated by connecting the spiral to an oscillatory circuit as in Fig. 132, where S is the secondary of a step-up transformer, C the condenser, L the inductance in the discharge circuit, G the spark gap, and o, m is a wire parallel to one side of the helix or spiral.

The dotted line o, a indicates shape and position of the potential wave. When the spiral is excited a fine

brush discharge may be seen filling the space between
the spiral and the wire parallel to it. When this brush
discharge is maximum at m and gradually decreases
as shown by the curve a, o, the wave is said to be the
fundamental. When the frequency of the e. m. f.
applied at e is increased, by decreasing the capacity
or inductance in the oscillatory circuit, the waves be-
come shorter until stationary waves of the first har-
monic are produced as shown by the line o, b. There
are two nodes and two antinodes, indicated by zero
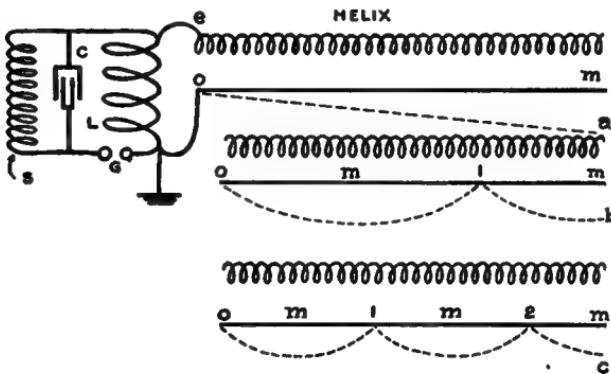
FIG. 132.

and maximum brush discharges, respectively. In-
creasing the frequency still further, three or more
nodes and antinodes can be created on the same wire,
as shown by the line o, c.

As one wave of e. m. f. has two antinodes and two
nodes, not including the node of origin, it is obvious
that, when the fundamental wave o, a is created, the
length of the spiral or helix is equal to one-quarter of
the wave length; in the first harmonic the wire in the
spiral is three-fourths of the wave length, in the sec-
ond harmonic five-fourths, etc.

It is clear from the foregoing that to obtain the maximum potential at terminal of a coil, grounded at one end, the length of the wire in the secondary coil should be one-quarter of the wave length of the oscillating e. m. f., or $\frac{3}{4}$, $\frac{5}{4}$, etc.

In a secondary coil where both ends are used for terminals the wire should be $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, etc., times the length of the e. m. f. wave, to make the antinodes of potential at the secondary terminals.

The effect produced by having resonance in circuits containing both capacity and inductance is probably best explained by analogy.

Consider a person sitting in a swing set in motion by another person standing on the ground. The swing has a natural time period determined by its length and the force of gravity. Now suppose that the person standing on the ground pushes the swing at regular intervals but of a different time period from that of the swing. It is evident that part of the time there will be opposition between the swing and the pushes, so that the swing will come to a stop or swing only intermittently, depending whether the difference in period between that of the swing and the pushes of the person doing the swinging is great or small. This compares with secondaries whose natural time periods do not harmonize with the periods of the primary oscillatory circuit.

The primary current tries to make the period of the secondary coil more like that of the primary, while the secondary current reacts upon the primary and tries to make its period closer to that of the secondary. If the difference in period is small, forced oscillations will be produced, but if the difference is great the secondary will hardly respond to the primary, no matter how much current is impressed on the primary circuit. If, however, the swing in the analogue is pushed at regular intervals having the same period as the natural

period of the swing, then the movement of the swing will be accelerated by every additional push until it might be dangerous to increase the amplitude of the swing.

Likewise a secondary, having the same natural time period as the current in the oscillatory circuit, can be made to yield enormous potentials between its terminals. The reactions between two tuning forks set some distance apart is also analogous to the reactions that take place between the primary and secondary of a high-frequency coil. Likewise, high potentials can be produced in circuits or coils, Fig. 132, by resonance, that are higher than the charging e. m. f. applied to one end of the circuit.

High-frequency transformers immersed in oil or petrolatum are very heavy and cumbersome, and since it is very difficult to make a wooden case perfectly oil-tight, air-insulated transformers have been successfully made to create very high potentials. Such coils are very light and can be carried about very conveniently.

The primary and secondary of an air-insulated high-potential coil can not be as closely coupled as when immersed in a better insulation, because the secondary discharge will jump across the short distance to the primary winding rather than the greater distance between the secondary terminals.

The coil in Fig. 130 can be used without oil or petrolatum, provided the condenser discharged through the primary is run at a lower potential, consequently the secondary potential of the coil will also be lower than when oil is used.

The best design of an air-insulated coil of high voltage must be such that will prevent a discharge between the primary and secondary and still have moderately close coupling. Close coupling has to be sacrificed to good insulation. The primary coils a, b, d, in Fig. 133,

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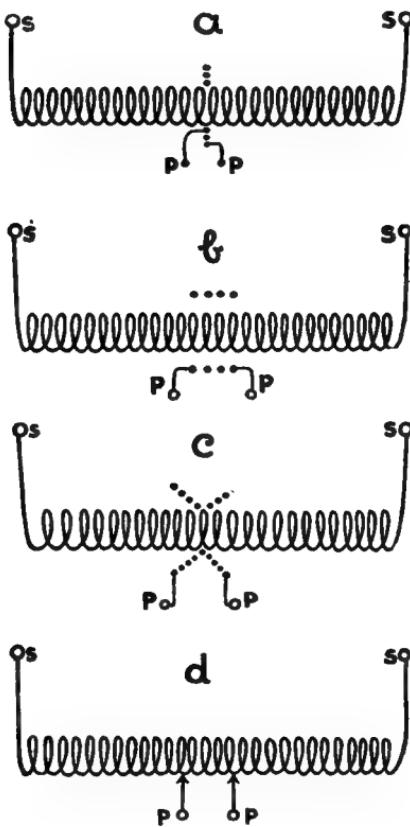
FIG. 133.

have splendid insulation from the high-potential por-
tions of the secondary, but have rather loose coupling.

The coil marked c has both close coupling and good
insulation space between the primary and the high-

potential portions of the secondary. The coil marked d is unique, in that the primary turns are common to both primary and secondary.

By splitting the primary and secondary coils in two parts, as in Fig. 134, the two sections can be set verti-
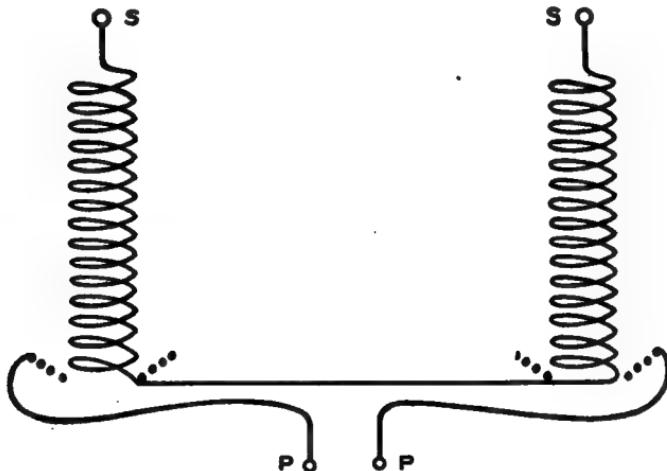
FIG. 134.

cally. This makes it possible to bring the terminals farther apart without as much danger of sparking to the primary and to the floor. Besides it is much more convenient to mount suitable terminals on the top of vertical sections than when the secondary is horizontal.

The primary may of course be made of the forms a, b, d, in Fig. 133, instead of the form c, if desired. The wire connecting the two primary sections can be used in common with the secondary, but must not be of any smaller wire than that composing the primary.

Both sections of this style of coil can be mounted on the ends of a long, flat board, which can be set upon

the condenser case when in use, thus making a light, compact bipolar coil capable of producing high potentials if windings are in the right proportions.

Each section of the coil can be used separately as

TACKS

WOODEN DISKS   PAPER
                        (PRESS BOARD)
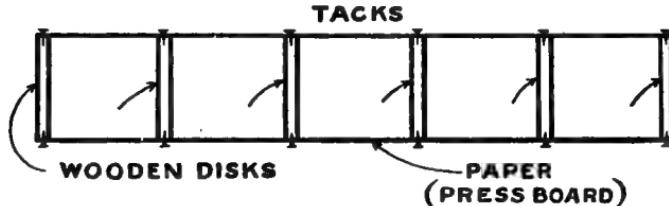
FIG. 135.

a unipolar transformer, popularly called an Oudin reso-nator (Fig. 135).

The coil f in Fig. 136 is connected to the condensers
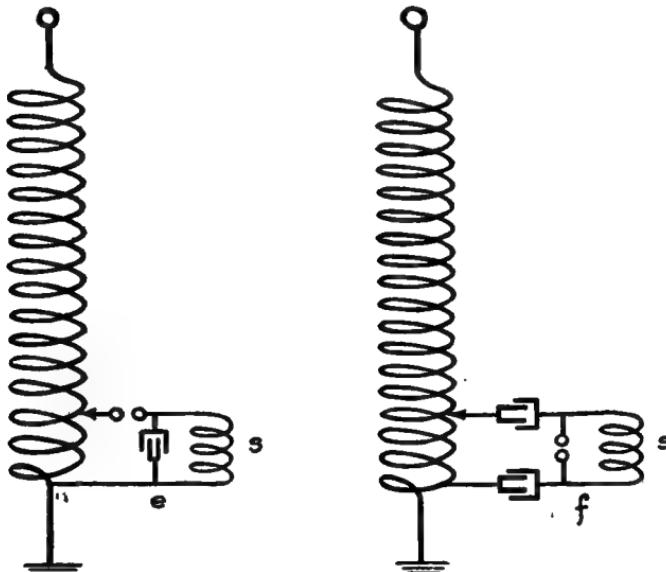
FIG. 136.

as it would be if the Leyden jars were charged by a static machine or an X-ray coil. The coil e is connected up the ordinary way to a step-up transformer.

A coil in which the primary is in common with the secondary possesses the advantage of having a secondary whose length can be changed by the sliding contact. This contact in moving up the side of the coil adds more turns to the primary and shortens the length of the secondary; both actions tend to bring the coil into tune.

When only a few of the lowest primary turns are used the frequency may be so high and the secondary coil so long that the top terminal may be a node of potential. When the contact is moved upward, more turns, and, consequently, more inductance are added. to the primary circuit. This lowers the frequency in the oscillatory circuit. With a shortened secondary and a lower frequency, which results in a longer wave length, the coil is brought into tune so that the top terminal of the secondary becomes an antinode.

The lower turns of the coil should be made from much larger wire than the upper turns, so as to offer as little resistance to the condenser discharge as possible.

Single-layer secondaries, such as described in Figs. 133, 134, 135, should be wound on paper tubes, made by tacking two or three thicknesses of moderately stiff pressboard on wooden disks spaced about 6 inches apart, as shown in Fig. 136. Tubes made from pieces of dry birchwood glued together also answer the purpose, but are harder to construct.

After the tube has been made by tacking the pressboard to the disks, another thickness of pressboard should be glued outside of the tube so as to cover up the heads of the tacks, which would otherwise prevent the even winding of the secondary layer.

Bare wire can be used in the secondary layer, pro-

vided adjacent turns are properly spaced apart. The spacing may be accomplished by running a sufficiently thick thread alongside of the wire while winding. The thread may be left in the coil. A neater way is to wind the coil in a lathe, properly spacing the turns, after which the layer is given one or two coats of orange shellac. This shellac gives the coil a good appearance and prevents the turns from sliding out of place. If a lathe can not be had, the turns may be spaced by hanging a small weight by a cord which is allowed to press against the side of the tube as the wire is being wound. The cord will move along sidewise with the wire and separate the adjacent turns the thickness of the cord. When the winding is complete the wire should be given a coat of shellac.

The compartment used to enclose the spark gap, used in the high-frequency experiments previously described, was made from a four-gallon crock jar fitted with an ebony asbestos cover (Fig. 137). A glass jar would have done just as well as the crock, and the cover could have been made from slate or glass with just as good results.

Into the cover was fitted a large insulating tube of hard rubber and into this tube was fitted a brass tube threaded to fit a long screw rod. Turning the rod by the hard-rubber handle on top of it gives a fine screw adjustment to the length of the spark gap.

The terminals of the spark gap were made from zinc battery rod cut off to the required length. Zinc is the metal best suited for spark-gap terminals to produce a sharp, snappy condenser discharge. The peculiar adaptability of zinc for this purpose ranks as high above that of other metals as the magnetic properties of iron ranks above all other substances. Brass is possibly next to zinc in efficiency because of the zinc it contains.

After a spark gap has been used for some time the

air in the compartment is burned up, causing the oxygen and nitrogen to chemically unite and form oxids of nitrogen. These oxids, with the moisture in the
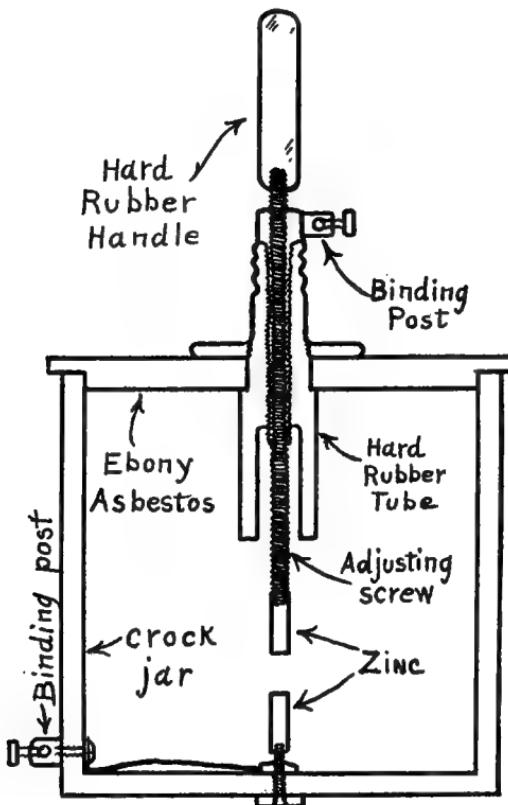
FIG. 137.

air, are deposited upon the inner surfaces of the jar, cover and insulating tube in the form of a conducting film. This film is detrimental to the proper working

of the spark gap, as it forms a shunt or by-path around the gap. The result is an inefficient and erratic discharge. To prevent this degeneracy the compartment must be cleaned frequently.

Although enclosing a spark gap in a jar with a loose cover smothers a great deal of the noise of the spark, it can not be entirely eliminated unless the gap is enclosed in a perfectly air-tight compartment having very thick walls. To prevent the troublesome oxids from forming, the compartment, after the air has been first exhausted from it, should be filled with nitrogen gas. When air or other gases are compressed they obtain a greater insulating strength; therefore a spark gap in a compressed gas must be shorter than when the gas is at atmospheric pressure. Yet the efficiency of this spark is higher because the breaking down of gas insulation is more sudden. Placing the gap in a uniform, powerful magnetic field also increases the efficiency of the spark gap.

As considerable power was required to produce the long discharges in Figs. 124, 125, 126 and 127, the primary of the step-up low-frequency transformer was run by 220 volts instead of the usual 110 volts. This was an extreme overload, but as the transformer was run only for a few seconds at a time, it stood the overload without a breakdown. Only half the primary turns were used, and only 220 turns of the reactance were connected in series with the primary. The yoke of the reactance coil was taken off entirely. This created a very high secondary voltage (about 35,000), which the transformer insulation withstood without a puncture.

When the transformer is run at this high voltage the brush discharge at the edge of the condenser coatings becomes excessive. A brush discharge is wasted energy, so it should be done away with. The easiest

way to do this is to immerse the entire condenser in insulating oil free from moisture and dirt.

When a condenser is immersed the dielectric punctures much easier than in open air, so it becomes necessary to use a thicker dielectric or else connect two condensers in series.

The condenser of nine Leyden jars previously described was used in series with the primary of the coil in Fig. 128 to produce the long discharges. If this condenser were unable to stand the e. m. f. of 35,000 volts, we find that when two such condensers are connected in series they can stand the pressure very easily. We find, however, that two condensers of the same size, connected in series, have only one-half the capacity of one condenser alone, thus two condensers of eighteen jars each must be connected in series to give the capacity of the nine original jars. Although four times as many jars are required when two condensers are connected in series, the series of jars will stand twice as high a voltage as the original nine.

When several condensers are connected in series the reciprocal of their combined capacity

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{ etc., thus } C = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \text{ etc.}}$$

where C_1, C_2, C_3, etc., are tne respective capacities of the condensers and C is the combined capacity.

When all the condensers connected in series are of the same capacity their combined capacity is equal to $\frac{\text{capacity of one}}{\text{number in series}}$. When two condensers of unequal capacity are connected in series the condenser of the smaller capacity will have the higher potential charge.

Leyden jars have very small capacity for the space they occupy, so that an enormous amount of oil would be required should the jars be immersed.

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One of the most convenient and very compact forms of high-voltage capacities is the adjustable-plate condenser shown in Fig. 138. A good quality of dou-
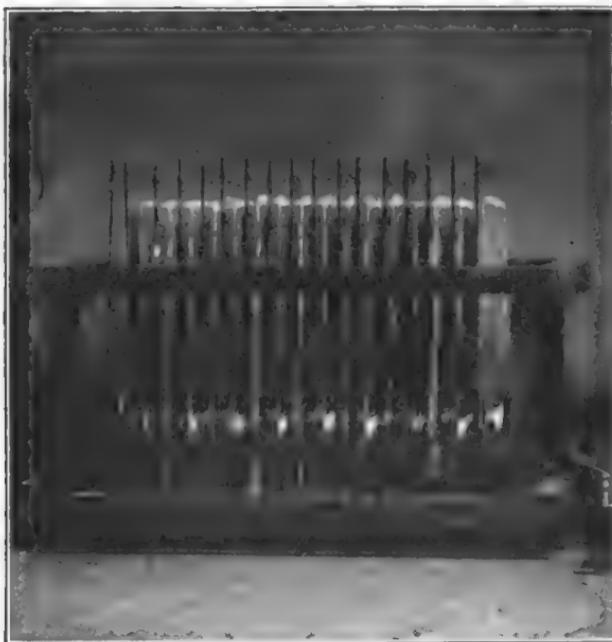
FIG. 138.

ble-thickness window glass answers most requirements for the dielectric if the e. m. f. does not exceed 10,000 volts.

Any required number of medium-size plates are coated on both sides with tin-foil as shown in Fig. 139 and then set on edge in a wooden crate (Fig. 140). A margin of 2 inches at the top and sides of the tin-foil

and of 5 inches at the bottom prevents discharge around the edge of the plate. Thin orange shellac is used to stick the tin-foil to the glass.

In the bottom of the crate and about 4 inches apart are two parallel rails of sheet copper (1½ inches wide) running the entire length of the crate. Binding-posts are attached to the ends of the rails for connection to other apparatus.

A strip of tin-foil 1½ inches wide extends down
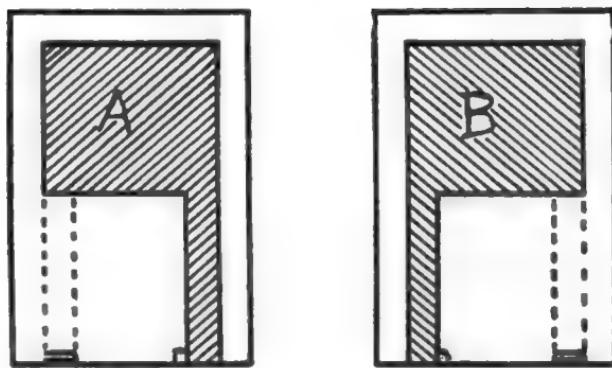
Fig. 139.

from one corner of the coating on each side of the glass to and slightly around the lower edge of the plate. These strips serve as contacts to the parallel copper rails upon which the plates rest. The strips and coating should be cut out of the tin-foil in one piece, as it is rather difficult to make a good contact between two pieces pasted together.

The tin-foil strip on one side of the plate connects with one of the copper rails, while the strip on the opposite side connects with the other rail. On half of the number of plates the strips should extend down

from the lower right-hand corner of the coating as on
Plate A in Fig 139, while on the rest of the plates the
tin-foil strip extends down from the lower left-hand
corner as on Plate B in Fig. 139.

Plates A and B are set in the crate in alternate
order, so that all the A plates are odd-numbered and
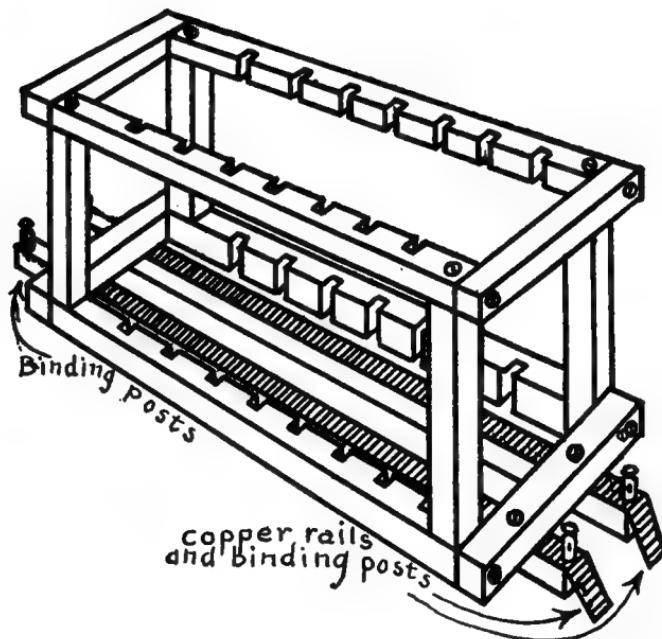
FIG. 140.

the B plates even-numbered. The object in this is to
have the coatings, that face each other on two adjacent
plates, of the same sign.

By adding plates or taking them out the capacity of
this condenser can be varied to meet one's require-

ments. The smaller the plates the finer will be the variation in capacity. For higher voltages than 10,000 the glass should be thicker and the margins wider. As stated before, it is better to connect two or more condensers in series to stand the increased e. m. f., for two thin dielectrics in series will stand a higher e. m. f. than one dielectric as thick as the two thin ones put together.

The plates of the condenser are set in vertical grooves, cut about an inch apart, in which they fit loosely, so that they can be easily set in or taken out. When the capacity is to be varied the plates should be added or taken out in consecutive order, with no vacant grooves between plates whose adjacent coatings are of the opposite sign, as there would be a tendency for a spark to jump between their facing coatings.

The proper and wrong ways to connect the condenser to the primary of the high-frequency coil are shown in Fig. 141. In diagram A it will be observed that the inductance in series with the condenser is the same for all the plates, but in B it is clear that the added inductance of the base rails is different for every plate, consequently close tuning becomes impossible.

A most compact form of high-voltage immersed condenser is shown in Fig. 142. Plates of glass or mica are stacked in a pile alternately with sheets of tin, which are cut in the same shape as the tin-foil coatings in Fig. 139. However, the margin on all sides need only be half as wide as in the open air. The strip contacts are made long enough to be soldered or screwed together to form terminals. In order to be certain that the oil in which the condenser is immersed will fill every portion of the space between the plates, the condenser should be assembled right in the oil. To keep the tin sheets and glass from sliding out of position, the condenser should be bound together

lengthwise and crosswise with a few turns of linen tape. If desired, melted petrolatum, paraffin or bees-wax can be used in place of the oil when the condenser is to be moved about a great deal. If oil is used, dou-ble-boiled linseed oil should be chosen.

Mica, or rather micanite, is much lighter than glass
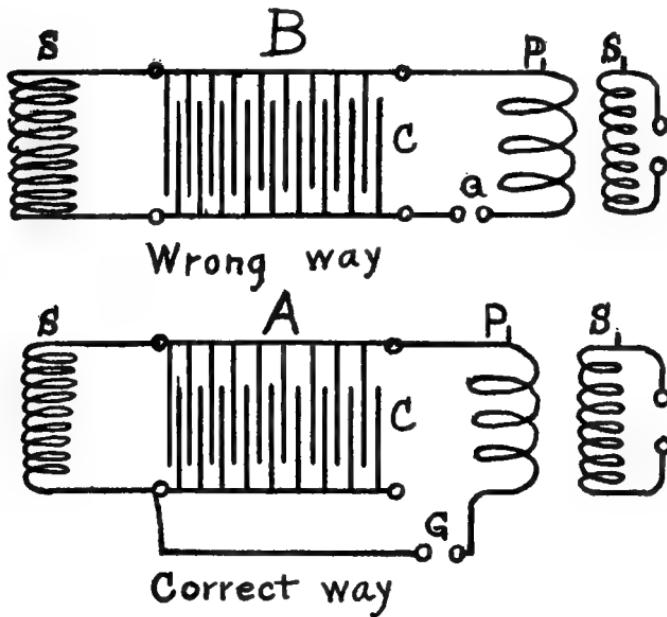
Fig. 141.

and does not break easily, therefore it is very well adapted for portable condensers. The insulation strength of micanite is greater than glass, consequently thinner plates can be used. Mica (or micanite) has one bad feature, however, which makes it unfit for some purposes, namely, that it is greatly affected by what is called dielectric hysteresis.

Dielectric hysteresis is due to causes similar to those of magnetic hysteresis, for both are due to the lagging of molecular rotation. The strain in a dielectric causes the molecules to whirl around and arrange themselves lengthwise along the electric lines of force just as magnetic iron molecules do when acted upon by magnetic lines of force.

The hysteresis or lagging of the molecules to turn themselves is caused by their crowded condition. As the molecules turn around they rub against other molecules. This friction produces heat in the dielectric, which lowers the insulation strength of the micanite to such an extent that it might be punctured. The heat produced represents a definite loss, which must be supplied by the charging current. One bad point about glass as a dielectric is that it becomes hard and brittle with age.

Since the two leading dielectrics have such bad features, successful attempts have been made to construct high-voltage air condensers. Air at atmospheric pressure has such a low dielectric strength that the condenser plates must be placed far apart to withstand high voltages. This makes the condenser of immense proportions, with only a small capacity. By compressing the air its dielectric strength can be increased so that it equals or exceeds that of glass. With air of higher insulating qualities the condenser plates may be brought much closer together without danger of breakdown. This makes possible a very compact condenser, with hysteresis and brush discharge practically eliminated. Of course, the specific inductive capacity of air is rather low, so that with the metallic condenser plates separated as far apart as in a condenser using glass as the dielectric, the air condenser would necessarily have to be from three to ten times as large as the glass condenser.

The capacity of plate condensers can be calculated

approximately by the following formula: The capacity in microfarads

$$C = \frac{\text{area of metallic coating (or plate) in sq. cm} \times K}{4\pi \times 900,000 \times d}$$

$$\text{or } C = \frac{\text{area in sq. cm.} \times K}{113,097,600 \times d}$$

where d is the number of centimeters the coatings (or tin plates) are apart and where K is the dielectric constant.

By using oil or compressed air (or other gas) as the dielectric the above formula is accurate enough for ordinary purposes, but when there is considerable brush discharge, as when glass or micanite plates are used in open air, the true capacity of the condenser is several per cent greater than the calculated capacity.

THE END.

ALPHABETICAL INDEX

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